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ABSTRACT

This is part two of a two-part SMSG elementary school text for fourth-grade students. The content is aimed at the development of some of the fundamental concepts of mathematics such as number, numeration, the operations of arithmetic, and intuitive geometry. Chapter topics include properties and techniques of addition and subtraction, techniques of multiplication and division, recognition of common geometric figures, linear measurement, and concept of rational numbers. (MP)

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ED173102

STUDENT'S TEXT

UNIT NO. 26

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# MATHEMATICS FOR THE ELEMENTARY SCHOOL

## GRADE 4

## PART II

SE 027 909

SCHOOL MATHEMATICS STUDY GROUP

YALE UNIVERSITY PRESS

2

# Mathematics for the Elementary School, Grade 4

## *Student's Text, Part II*

REVISED EDITION

Prepared under the supervision of the
Panel on Elementary School Mathematics
of the School Mathematics Study Group:

| | |
|---|---|
| Leslie Beatty | Chula Vista City School District,<br>Chula Vista, California |
| E. Glenadine Gibb | Iowa State Teachers College,<br>Cedar Falls, Iowa |
| W. T. Guy | University of Texas |
| S. B. Jackson | University of Maryland |
| Irene Sauble | Detroit Public Schools |
| M. H. Stone | University of Chicago |
| J. F. Weaver | Boston University |
| R. L. Wilder | University of Michigan |

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## Chapter 6

# PROPERTIES AND TECHNIQUES OF ADDITION AND SUBTRACTION II

## DIFFERENT NAMES FOR THE SAME NUMBER

There are many ways of naming a number. The decimal numeral for 40 + 2 is 42. It may also be named in other ways.

1. Nan says that all the names below are for the same number. Do you agree? 27 - 3; 24 + 0; 10 + 14; 25 - 1; 2 tens and 4; 1 ten and 14.

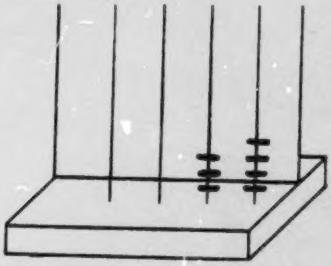
2. What is the decimal numeral for 40 + 15? State five other names for 40 + 15.

3. (a) Is 234 = 200 + 30 + 4? (c) Is 234 = 200 + 20 + 14?
   (b) Is 234 = 200 + 10 + 24? (d) Is 234 = 100 + 130 + 4?

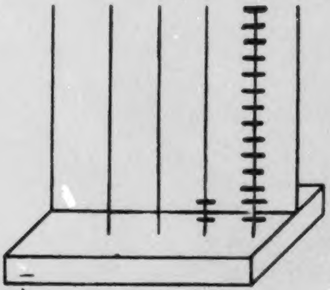
4. You may think of 67 as 6 tens and 7 ones or as 5 tens and 17 ones. What are other names for 67?

   May we think of 726
   as 700 + 20 + 6?
   as 700 + 10 + 16?
   as 600 + 120 + 6?

   Different names for a number are often shown on an abacus. How is 34 named on each abacus at the right?



34 as 3 tens
and 4 ones



34 as 2 tens
and 14 ones

5. Tell two different names for each of these numbers. Show each on the abacus.
   (a) 46 (b) 97 (c) 263

## Exercise Set 1

Copy the numerals 1 - 10 on your paper. Next to each write the correct answers to complete this chart.

| | Decimal Numeral | Another Name for the Number |
|---|---|---|
| 1 | _____ | 120 + 17 |
| 2 | _____ | 1200 + 160 + 18 |
| 3 | 523 | _____ |
| 4 | _____ | 6 tens + 18 |
| 5 | 96 | _____ |
| 6 | _____ | 4 hundreds + 15 tens + 8 |
| 7 | 238 | _____ |
| 8 | 49 | _____ |
| 9 | _____ | 15 hundreds + 23 tens + 19 |
| 10 | 1,526 | _____ |

For each of exercises 11 - 13 write > or < so each mathematical sentence will be true. In exercise 11, is 1000 + 300 + 60 + 16 another name for 1376?

11. 1,378 _____ 1000 + 300 + 60 + 16

12. 2,874 _____ 1 thousand + 17 hundreds + 16 tens + 4 ones

13. 4,926 _____ 3 thousands + 18 hundreds + 11 tens + 6 ones

## Exercise Set 2

1. A salesman traveled 453 miles in January and 523 miles in February. What distance did he travel in the two months?

2. The salesman traveled 230 miles in March, 310 miles in April, and 345 miles in May. How many miles did he travel in the three months?

3. From January through June the salesman traveled 2,010 miles. From July through December he traveled 1,854 miles. How far did he travel during the year?

4. You found how far the salesman traveled in one year in exercise 3. During another year he traveled 4,013 miles. What was his mileage during the two years?

5. On an automobile trip, Fred and Carol played a game by counting station wagons and trucks they saw on the highway. Fred counted 234 station wagons and Carol counted 205 trucks. How many station wagons and trucks did they count in all?

6. Jack and Tim have been gathering rocks for the new walk their father is making. Jack has gathered 172 rocks and Tim has gathered 213. How many rocks have the two boys gathered altogether?

## MORE ADDITION

1. What number is n if 423 + 345 + 214 = n?

First, place beads on the abacus to show the addends so that each addend is separated from the others.

$$\begin{array}{r} 400 + 20 + 3 \\ 300 + 40 + 5 \\ \underline{200 + 10 + 4} \end{array}$$

Next, show the result of adding the ones.

Show the result of adding the tens.

Show the result of adding the hundreds.

$$\begin{array}{r} 400 + 20 + 3 \\ 300 + 40 + 5 \\ \underline{200 + 10 + 4} \\ 900 + 70 + 12 \end{array}$$

Now 423 + 345 + 214 = 900 + 70 + 12.

900 + 70 + 12 is thought of as 900 + 70 + (10 + 2).

900 + 70 + (10 + 2) = 900 + (70 + 10) + 2.

What is the decimal numeral for 900 + 80 + 2?

2. Now try to add 342, 124 and 418 without the abacus. See **Box A.**

a. What numbers were added first?

b. What decimal numeral is 800 + 70 + 14?

$$\boxed{\begin{array}{rcl} 342 & = & 300 + 40 + \phantom{1}2 \\ 124 & = & 100 + 20 + \phantom{1}4 \\ \underline{418} & = & \underline{400 + 10 + \phantom{1}8} \\ & & 800 + 70 + 14 = 884 \end{array}}$$

A

3. Find n if 375 + 278 = n. You might try the method on page 306. In Box B, the decimal numeral 653 was obtained from adding 500, 140, and 13.

$$\boxed{\begin{array}{l} 375 = 300 + 70 + 5 \\ 278 = \underline{200 + 70 + 8} \\ \qquad\;\; 500 + 140 + 13 = 653 \end{array}}$$

B

4. Sometimes 375 and 278 are added as in box C. What numbers were added to get 13? How do you get the 140? How do you get 500? How is the 653 obtained? The method of Box C may be more convenient for you.

$$\boxed{\begin{array}{r} 375 \\ \underline{278} \\ 13 \\ 140 \\ \underline{500} \\ 653 \end{array}}$$

C

## Exercise Set 3

1. Use the method of Box B to find each sum.

$$\begin{array}{r} 43 \\ \underline{29} \end{array} \qquad \begin{array}{r} 167 \\ \underline{254} \end{array} \qquad \begin{array}{r} 346 \\ \underline{186} \end{array} \qquad \begin{array}{r} 558 \\ \underline{645} \end{array} \qquad \begin{array}{r} 1287 \\ \underline{3648} \end{array}$$

2. Use the method of Box C to find each sum.

$$\begin{array}{r} 429 \\ \underline{385} \end{array} \qquad \begin{array}{r} 697 \\ \underline{134} \end{array} \qquad \begin{array}{r} 1278 \\ \underline{4193} \end{array} \qquad \begin{array}{r} 8296 \\ \underline{1376} \end{array} \qquad \begin{array}{r} 6278 \\ \underline{1032} \end{array}$$

## ANOTHER METHOD FOR ADDING

Addition is an operation on two numbers. When we operate on 15 and 3 and get 18, we have added. (15 + 3 = 18) Eighteen is called the sum. Fifteen and 3 are called addends.

An addition exercise is written in columns to make it easy to add. Columns help to keep the ones together, the tens together, the hundreds together, and so on.

In column addition the ones are added first, the tens next, the hundreds next, and so on.

Part of the sum of the ones' column is sometimes remembered. It is then added in with the tens.

Part of the sum of the tens' column is sometimes remembered. It is then added in with the hundreds.

$$\begin{array}{r}329\\146\\\underline{948}\end{array}$$

To add I think: 9 and 6 are 15 and 15 and 8 are 23. Think of 23 as 2 tens and 3 ones. Record 3 and remember 2 tens.

Two tens and 2 tens are 4 tens; 4 tens and 4 tens are 8 tens; and 8 tens and 4 tens are 12 tens. Think of 12 tens as 1 hundred and 2 tens. Record 2 tens and remember 1 hundred.

One hundred and 3 hundreds are 4 hundreds; 4 hundreds and 1 hundred are 5 hundreds; and 5 hundreds and 9 hundreds are 14 hundreds. Record 14 hundreds.

## Exercise Set 4

Find the sums for exercises 1 through 5.

| | (a) | (b) | (c) | (d) | (e) | (f) |
|---|---|---|---|---|---|---|
| 1. | 43<br>29 | 57<br>38 | 19<br>46 | 76<br>15 | 68<br>28 | 53<br>17 |
| 2. | 126<br>246 | 348<br>629 | 167<br>726 | 239<br>43 | 468<br>504 | 282<br>509 |
| 3. | 563<br>128 | 635<br>406 | 447<br>129 | 563<br>129 | 38<br>257 | 647<br>39 |
| 4. | 174<br>138 | 88<br>543 | 489<br>272 | 179<br>658 | 266<br>698 | 593<br>248 |
| 5. | 347<br>897<br>304 | 256<br>1297<br>540<br>698 | 1591<br>8643<br>9275<br>5873 | 1876<br>7235<br>8544<br>6718 | 8976<br>1235<br>7142<br>6473 | 1762<br>4391<br>3065<br>8572 |

6. Find n for each of exercises (a) through (d).

(a) $n = 697 + 384$

(b) $n = 672 + 1278$

(c) $n - 559 = 2476$

(d) $362 = n - 875$

## Exercise Set 5

1. List the number of days in each of the first six months of this year. How many days are there in the first six months of this year?

2. List the number of days in each of the last six months of this year. How many days are there in the last six months of this year?

3. John went to a book store. He found 5 magazines which he wanted. Their prices were 75¢, 20¢, 25¢, 55¢, and 95¢. He bought the three which were cheapest. How much did they cost?

4. There were 135 books borrowed from the library on Monday, 140 books on Tuesday, 168 books on Wednesday, 174 books on Thursday, and 147 books on Friday. During these five days, how many books were borrowed?

5. The Jackson family took a trip by car from New York City to Boston. The trip took five hours. This is how far they traveled each hour: 36 miles, 44 miles, 47 miles, 41 miles, and 38 miles. How many miles did they travel in the five hours?

6. John's mother bought him a new coat, cap, shoes, and boots. The cost of the coat was $18, the cap $3, the shoes $8, and the boots $6. How much did she pay for them all?

7. There are 65,761 Indians in Arizona, 53,769 Indians in Oklahoma, and 41,901 Indians in New Mexico. How many Indians live in these three states?

8. There are 629 boys and 587 girls in Longfellow School. How many children attend Longfellow School?

9. In 1940 there were 172,172 people in Miami, Florida. In 1950 there were 87,063 more people living there than in 1940. How many people lived in Miami in 1950?

10. During a candy sale Mary sold 232 boxes of mints. Sue sold 472 boxes, and Jane sold 143 boxes. Find the total number of boxes sold by the three girls.

11. The pupils of Oak School collected gifts for poor children at Christmas. They collected 433 books, 316 toys, 252 games, and 164 puzzles. How many gifts were collected in all?

16

## Exercise Set 6

Copy the numerals 1 through 8 on your paper. Next to each numeral write the words and numerals to complete this chart.

| | Numbers Operated On | Result | Operation Used | Mathematical Sentence |
|---|---|---|---|---|
| 1. | 15, 289 | ____ | addition | ____________ |
| 2. | 139, 76 | 215 | addition | ____________ |
| 3. | | n | addition | 674 + 879 = n |
| 4. | 71, 56 | 127 | ________ | ____________ |
| 5. | 641, (379 + 81) | ____ | addition | ____________ |
| 6. | 162, 69 | ____ | addition | ____________ |
| 7. | 345, 187 | 532 | ________ | ____________ |
| 8. | 647, 387 | ____ | addition | ____________ |

Write = or ≠ so each of exercises 9 through 15 will be true mathematical sentences.

9. 372 + 499 _____ 773

10. 312 + 184 _____ 128

11. 346 + n _____ 179, if n = 177

12. n + 156 _____ 394, if n = 328

13. n - 341 _____ 159, if n = 500

14. If n = 379, then n + 172 _____ 308 + 233

15. If n = 473, then 896 + n _____ 674 + 595

## REVIEW OF SUBTRACTION

### Exercise Set 7

1. George had 524 for the answer to an exercise. It should have been 639. How much too small was his answer?

2. The zoo keeper told Jim that the big gorilla weighed 572 pounds, and the small one weighed 361 pounds. How much more does the large gorilla weigh?

3. In 1950, the population of a city was 6,478. By 1960, it had increased to 9,699. What was the increase in population during the ten-year period?

4. The Boy Scouts had a paper drive. Troop 51 collected 8,200 pounds of paper. They wanted to collect 9,600 pounds. How many more pounds of paper do they need to collect?

5. Subtract

| | | | |
|---|---|---|---|
| 665 | 841 | 937 | 269 |
| 152 | 721 | 125 | 253 |

6. Find n so each mathematical sentence will be true.

(a) $n + 395 = 697$ (c) $863 + n = 1175$

(b) $n = 1158 - 737$ (d) $2378 - 2163 = n$

MORE SUBTRACTION

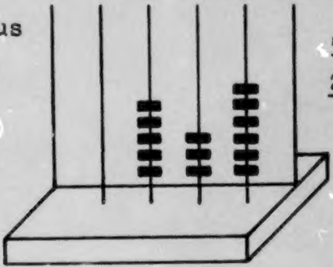
Subtraction is an operation for finding the unknown addend if the sum and one addend are known. To find 536 - 218, you have learned to write as shown in box A.

A 536 sum
218 addend
? unknown addend

1. Could you subtract using the form of box B? Why?

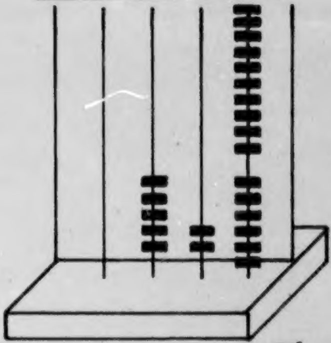
B $536 = 500 + 30 + 6$
$\underline{218} = \underline{200 + 10 + 8}$

Now, let us use the abacus to help us think about this process. First we show the sum 536 on the abacus



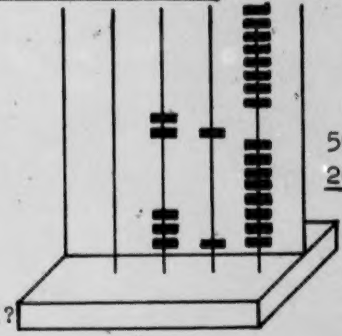
$500 + 30 + 6$
$\underline{200 + 10 + 8}$

Then, we think of the sum 536 as $500 + 20 + 16$



$500 + 20 + 16$
$\underline{200 + 10 + 8}$

Now, we separate the markers to show the known addend, 218, and the other addend. What is the other addend?



$500 + 20 + 16$
$\underline{200 + 10 + 8}$

2. The written record of the above subtraction is

$$\begin{array}{lclcl}
536 & = & 500 + 30 + 6 & = & 500 + 20 + 16 \\
\underline{218} & = & \underline{200 + 10 + 8} & = & \underline{200 + 10 + \ \ 8} \\
 & & & & 300 + 10 + \ \ 8 \ = \ 318
\end{array}$$

3. Sometimes, finding the unknown addend is more difficult.

For example, what is $n$, if $268 + n = 932$?

We may write:

$$\begin{array}{lclclcl}
932 & = & 900 + 30 + 2 & = & 900 + 20 + 12 & = & 800 + 120 + 12 \\
\underline{268} & = & \underline{200 + 60 + 8} & = & \underline{200 + 60 + \ \ 8} & = & \underline{200 + \ \ 60 + \ \ 8} \\
 & & & & & & 600 + \ \ 60 + \ \ 4 = 664
\end{array}$$

Explain how we may think when subtracting in this way.

What is the other addend?

**Now let us look for a shorter way of writing the steps in a subtraction problem. Notice how this form corresponds to the one above. We begin with**

$$\begin{array}{r}
932 \\
\underline{-\ 268}
\end{array}
\qquad
\begin{array}{l}
\textbf{9 hundreds, 3 tens, 2 ones} \\
\textbf{2 hundreds, 6 tens, 8 ones.}
\end{array}$$

**We cannot subtract in the ones' column so we regroup**

$$\begin{array}{ccc}
 & ② & ⑫ \\
9 & \cancel{3} & 2 \\
2 & 6 & 8
\end{array}
\qquad
\begin{array}{l}
\textbf{9 hundreds, 2 tens, 12 ones} \\
\textbf{2 hundreds, 6 tens, 8 ones.}
\end{array}$$

**We cannot subtract in the tens' column so we regroup again**

$$\begin{array}{ccc}
⑧ & ⑫ & ⑫ \\
\cancel{9} & \cancel{3} & \cancel{2} \\
\underline{-\ 2} & \underline{6} & \underline{8} \\
6 & 6 & 4
\end{array}
\qquad
\begin{array}{l}
\textbf{8 hundreds, 12 tens, 12 ones} \\
\textbf{2 hundreds, 6 tens, 8 ones}
\end{array}$$

**$12 - 8 = 4$, 4 ones**

**$12 - 6 = 6$, 6 tens**

**$8 - 2 = 6$, 6 hundreds**

## ANOTHER METHOD FOR SUBTRACTING

Subtraction is an operation on numbers. When we operate on 15 and 3 and get 12, we have subtracted. $15 - 3 = 12$. And, 12 is called the unknown addend.

A subtraction exercise is written in columns to make subtraction easy. Columns help to keep the ones together, the tens together, etc.

In column subtraction the ones are subtracted first, the tens next, etc.

Renaming the sum in a subtraction exercise may help us to subtract.

$$\begin{array}{r} 5576 \\ -\ \underline{1328} \end{array}$$

To subtract I think:

There are not enough ones in the ones' place in 5,576. I will think of 5,576 as 5 thousands, 5 hundreds, 6 tens, and 16 ones. $16 - 8 = 8$. $6 - 2 = 4$. $5 - 3 = 2$. $5 - 1 = 4$. The unknown addend is 4,248.

### Exercise Set 8

Find the unknown addend in each of 1 and 2.

| | (a) | (b) | (c) | (d) | (e) |
|---|---|---|---|---|---|
| 1. | 93 | 187 | 817 | 852 | 596 |
| | 38 | 99 | 748 | 575 | 378 |
| 2. | 5634 | 2376 | 8421 | 3124 | 5672 |
| | 1256 | 259 | 5167 | 2674 | 1489 |

## Exercise Set 9

1. One week a factory assembled 2,640 trucks and 1,582 automobiles. How many more trucks than automobiles were assembled?

2. In 1950 there were 3,500 people in Woodside. In 1960 there were 9,400 people in Woodside. How many more people were there in 1960 than in 1950?

3. We planned a 455 mile trip. The first day we traveled 266 miles. How many miles were left to travel?

4. The Mississippi River is 2,348 miles long and the Ohio River is 981 miles long. How many miles longer is the Mississippi River?

5. What is the total length of the Mississippi and the Ohio Rivers?

6. In New York City, the Empire State Building is 1,472 feet high. The Chrysler Building is 1,046 feet high. How much higher is the Empire State Building?

7. There were 435 children at Whittier School and 379 children at Edison School. How many children attend both schools?

8. A sign on a foot bridge reads, "Not safe for over 200 pounds." Jerry weighs 62 pounds, Dick weighs 57 pounds, Tom weighs 68 pounds. Can the three boys safely walk across the bridge together?

9. Another bridge holds two tons safely. A cement truck that weighs 2,165 pounds is on the bridge. How many more pounds could safely be on the bridge at the same time?

10. Susan's grandmother was born in 1908. How old will she be on her birthday this year?

## SUBTRACTION WITH ZEROS

1. Which of these are other names for 8,000?

| | | |
|---|---|---|
| (a) 8,000 ones | (d) 7,000 + 1,000 | (g) 8 thousands |
| (b) 8021 - 21 | (e) 800 hundreds | (h) 8,000 - 0 |
| (c) 800 tens | (f) 10,000 - 2,000 | |

2. Suppose you are to find n when 8,000 - 1,732 = n. You can write the example as in **Box A**. Finding the unknown addend is easy if you rename 8,000 as 799 tens and 10 ones or 7990 + 10.

A

$$\begin{array}{r} 8000 \\ \underline{-\ 1732} \end{array}$$

3. (a) Look at the example given in **Box B.**

   (b) Tell how to get the unknown addend, 6260 + 8.

   (c) What decimal numeral names the unknown addend?

B

$$\begin{array}{rcr} 8000 & = & 7990 + 10 \\ \underline{1732} & \underline{=} & \underline{1730 + \ \ 2} \\ & & 6260 + \ \ 8 \end{array}$$

### Exercise Set 10

Find the unknown addend for each of these.

| | (a) | (b) | (c) | (d) |
|---|---|---|---|---|
| 1. | $\begin{array}{r} 804 \\ \underline{267} \end{array}$ | $\begin{array}{r} 602 \\ \underline{536} \end{array}$ | $\begin{array}{r} 102 \\ \underline{85} \end{array}$ | $\begin{array}{r} 3001 \\ \underline{1467} \end{array}$ |
| 2. | $\begin{array}{r} 6000 \\ \underline{1234} \end{array}$ | $\begin{array}{r} 3007 \\ \underline{1562} \end{array}$ | $\begin{array}{r} 4803 \\ \underline{1297} \end{array}$ | $\begin{array}{r} 2067 \\ \underline{1982} \end{array}$ |

## Exercise Set 11

1. John is 52 inches tall; his father is 70 inches tall. How many inches must John grow to be as tall as his father?

2. At Glenn School there are 500 boys and 375 girls. How many more boys are there than girls?

3. Don has 1,500 stamps. He pasted 323 in his album. How many are left to put in the album?

4. Sue has $25. She is saving to buy a bicycle which costs $42. How much more money must she save?

5. A high school stadium has 5,200 seats. 3,482 tickets have been sold for a game. How many tickets are left?

6. An elephant in a zoo weighs 5,000 pounds. A bear weighs 746 pounds. How much less does the bear weigh than the elephant?

7. West Virginia became a state in 1863. Hawaii became a state in 1960. How many more years has West Virginia been a state than Hawaii?

## RELATION OF THE TECHNIQUES OF ADDITION AND SUBTRACTION

### Exercise Set 12

Copy the chart below. Add or subtract each exercise and then undo each.

| Do | | | Undo |
|---|---|---|---|
| 1. | Add | 725<br>342 | 1. |
| 2. | Subtract | 1629<br>817 | 2. |
| 3. | Subtract | 5232<br>768 | 3. |
| 4. | Add | 5287<br>9388 | 4. |
| 5. | Add | 26534<br>12986 | 5. |

6. Show that each of these mathematical sentences about doing and undoing is true. The first one is done for you as an example.

(a) (573 + 128) - 128 = 573.

Answer:
$$\begin{array}{r} 573 \\ \underline{128} \\ 701 \end{array} \qquad \begin{array}{r} 701 \\ \underline{128} \\ 573 \end{array}$$

(b) (841 + 368) - 368 = 841
(c) (632 - 257) + 257 = 632
(d) (905 - 496) + 496 = 905
(e) (384 + 769) - 769 = 384

7. Column addition may be checked by using the commutative and associative properties of addition. In this example, first "add from the top down." Then "add from the bottom up." Are the sums the same? Add:

$$\begin{array}{r}43\\32\\\underline{57}\end{array}$$

Add and check the sums in each of the following exercises:

8. $$\begin{array}{r}72\\49\\36\\42\\\underline{88}\end{array}$$

9. $$\begin{array}{r}324\\964\\322\\\underline{508}\end{array}$$

10. $$\begin{array}{r}3286\\9246\\3078\\\underline{5000}\end{array}$$

11. $$\begin{array}{r}17208\\15363\\\underline{42630}\end{array}$$

12. $$\begin{array}{r}1492\\3876\\9547\\3841\\\underline{2056}\end{array}$$

13. $$\begin{array}{r}687\\941\\600\\817\\\underline{932}\end{array}$$

14. $$\begin{array}{r}15618\\29832\\75490\\61078\\\underline{70201}\end{array}$$

15. $$\begin{array}{r}61429\\78503\\59268\\68107\\\underline{91030}\end{array}$$

16. BRAINTWISTER: **Try to find the sum for exercise 8 by adding down the column once.**

## Exercise Set 13

Copy the numerals 1 through 7 on your paper. Write the correct words or numerals to complete this chart.

| | Numbers Operated On | Result | Operation | Mathematical Sentence |
|---|---|---|---|---|
| 1. | 394, 869 | ___ | addition | ___ |
| 2. | ___, ___ | ___ | ___ | 762 - ___ = 575 |
| 3. | 498, ___ | 1277 | addition | ___ |
| 4. | (297 + 356), 495 | ___ | subtraction | ___ |
| 5. | 2000, (156 + 354) | ___ | subtraction | ___ |
| 6. | (392 + 867), 201 | ___ | subtraction | ___ |
| 7. | ___, ___ | ___ | ___ | 384 + 979 = ___ |

**In exercises 8 to 16, what is n so each mathematical sentence will be true?**

8. n = 67 + 48
9. n = 204 - 157
10. n = 4000 - 1963
11. n + 42 = 89
12. 102 - n = 3
13. n - 128 = 568
14. n + 392 = 691
15. 601 - n = 399
16. 893 - n = 256

17. **BRAINTWISTER.** In each exercise below, the letters A, B, C, D and E are to be replaced by one of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 or 9. They may be replaced by different digits in different exercises. A symbol such as AB represents a 2-place numeral.

$$\begin{array}{r} 47 \\ +\ \ 4 \\ \hline A1 \end{array} \qquad \begin{array}{r} 63 \\ -\ \ A \\ \hline B8 \end{array} \qquad \begin{array}{r} 47 \\ +\ \ D \\ \hline CC \end{array} \qquad \begin{array}{r} DD \\ +\ \ E \\ \hline 80 \end{array} \qquad \begin{array}{r} CC \\ +\ \ C \\ \hline 60 \end{array}$$

$$\begin{array}{r} ABC \\ -\ \ 42 \\ \hline 153 \end{array} \qquad \begin{array}{r} E4 \\ +\ 3C \\ \hline 87 \end{array} \qquad \begin{array}{r} BB \\ -\ B2 \\ \hline 4 \end{array} \qquad \begin{array}{r} 7A \\ -\ AB \\ \hline 14 \end{array} \qquad \begin{array}{r} CD \\ +\ D3 \\ \hline 79 \end{array}$$

## THE LANGUAGE OF SUBTRACTION PROBLEMS

1. Family A traveled 323 miles and family B traveled 289 miles on a weekend trip. How many more miles did family A travel than family B?

2. Family A traveled 323 miles and family B traveled 289 miles on a weekend trip. How far did the two families travel?

3. Family A traveled 323 miles and family B traveled 289 miles on a weekend trip. How much farther would family B have to travel in order to travel as far as family A?

4. Families A and B are together on a trip of 323 miles. They have traveled 289 miles. How many miles do they have left to travel?

### Exercise Set 14

1. A notebook costs 15¢, a pencil 27¢, and an eraser 5¢. How much will it cost to buy a set of one of each?

2. Four children put their savings together to help buy a riding horse. Mary had $35, Jerry had $48, Diane had $123, and Frank had $97. How much money did the four children have?

3. A football playing field is 300 feet long and 160 feet wide. How far will you have walked if you walk along the four edges of the field?

4. John has 268 postage stamps. He received some for Christmas. Then he had 323. How many stamps did he receive for Christmas?

5. At Fairview, the temperature was 58° at noon and 23° at midnight. How much had the temperature changed?

6. Tom wanted to buy a radio which was priced at $72. He had $56 saved. How much did he still have to save?

7. On a page in a catalog the following prices were given: soft ball, $1; bat, $3; fielder's mitt, $3; catcher's mitt, $12; first baseman's mitt, $9; catcher's mask, $4; and baseball uniform, $6. What will it cost Mr. Thompson to buy a ball, a bat, and three uniforms for his sons?

8. In one year the Acme Motor Company made 969,732 automobiles, 95,060 trucks, and 17,747 motor scooters. Find the number of vehicles made by the Acme Motor Company in that year.

## IF-THEN THINKING

1. We often use "if-then" reasoning. For example, you may think:

   "If I run home, then I will get there quicker." or

   "If it rains, then we cannot play baseball."

   Tell some "if-then" statements about your activities.

2. In our if-then statements we want the second part to be true because of the first part.

3. We use "if-then" thinking when we reason:

   "If $7 + n = 15$, then $n + 7 = 15$" or

   "If $7 + n = 15$, then $n = 8$".

   We would not think

   "If $3 + 6 = 9$, then $3 + 6 = 10$, since the "then" part is not a result of the "if" part.

   We could think, "If $3 + 6 = 9$, then $3 + 7 = 10$."

   Complete this statement in other ways, If $3 + 6 = 9$, then . . .

4. (a) Is it true that "If $n + 6 = 15$, then $n = 15 - 6$"?

   (b) Is it true that "If $n - 6 = 10$, then $n = 10 + 6$"?

## Exercise Set 15

1. Complete these statements. Use some different ways to complete each as you can.

(a) If $15 - 9 = n$, then ...

(b) If $13 + n = 21$, then ...

(c) If $33 = 17 + n$, then ...

(d) If $11 + n = 25$, then ...

(e) If $12 + n = 19$, then ...

(f) If $n - 15 = 14$, then ...

2. Use $=$, $>$ or $<$ so each of these statements will be true.

(a) If $n + 6 = 17$, then $n$ ____ $17$.

(b) If $21 - n = 19$, then $n$ ____ $21$.

(c) If $44 = n + 27$, then $n$ ____ $44$.

(d) If $n - 16 = 31$, then $n$ ____ $16$.

(e) If $n + n = 40$, then $n$ ____ $40$.

(f) If $n + 0 = 178$, then $n$ ____ $178$.

(g) If $0 - n = 0$, then $n$ ____ $0$.

(h) If $(6 + 8) + n = 19$, then $n$ ____ $19$.

3. BRAINTWISTER. Remember: $x$, $y$, and $z$ represent whole numbers. Suppose $x + y = z$.

(a) Are you sure that $x < z$ and $y < z$?

(b) Give an example for $x = z$.

(c) Give an example for $x < z$.

(d) Give one example for $x < z$ and $y < z$.

(e) Could $x > z$?

## Exercise Set 16

The arrangement of numbers in the square at the right is called a magic square.

| | A | B | C |
|---|---|---|---|
| D | 4 | 3 | 8 |
| E | 9 | 5 | 1 |
| F | 2 | 7 | 6 |

1. What is the sum of the numbers in column A? in column B? in column C?

2. What is the sum of the numbers in row D? row E? row F?

3. The 4, 5, and 6 are said to be on a diagonal. What is their sum? What three other numerals are on a diagonal? What is their sum?

4. Are all eight sums the same? The square is said to be "magic" because the sums of all rows, columns, and diagonals are equal.

5. Make a new square by adding 19 to each number in the above square. What is the sum of the numbers in: each row? each column? each diagonal? Is the new square a magic square?

6. Is the square on the right a magic square? What is the sum of the numbers on each row, column, and diagonal?

| | | | |
|---|---|---|---|
| 71 | 57 | 58 | 68 |
| 60 | 66 | 65 | 63 |
| 64 | 62 | 61 | 67 |
| 59 | 69 | 70 | 56 |

7. Make a new square by subtracting 49 from each number in the square in exercise 6. Is it a magic square?

REVIEW

## Exercise Set 17

1. Subtract

$$\begin{array}{rrrrr} 84 & 126 & 536 & 1427 & 1674 \\ \underline{57} & \underline{67} & \underline{239} & \underline{1148} & \underline{555} \end{array}$$

2. Add

$$\begin{array}{rrrrr} 67 & 134 & 257 & 3782 & 2841 \\ \underline{84} & \underline{29} & \underline{489} & \underline{6356} & \underline{7159} \end{array}$$

3. Find $n$ so each mathematical sentence will be true.

(a) $81 - 46 = n$ (c) $n + 126 = 253$ (e) $359 - n = 284$

(b) $n = 76 + 49$ (d) $n - 87 = 123$ (f) $283 + n = 481$

4. Which of these mathematical sentences are not true?

(a) $81 + 69 = 160$ (d) $1276 - 493 = 783$

(b) $124 + 238 = 362$ (e) $263 = 612 - 350$

(c) $289 + 463 = 752$ (f) $412 = 913 - 571$

5. Write $=$, $>$ or $<$ so each mathematical sentence will be true.

(a) 825 ____ 568 + 257 (c) 742 - 367 ____ 374

(b) 289 + 482 ____ 761 (d) 538 - 289 ____ 259

6. **BRAINTWISTER.** What whole number, if any, can be used for n so each mathematical sentence will be true?

(a) $192 + n = 168$

(b) $192 + n = 268$

(c) $312 - n = 314$

(d) $312 - n = 310$

(e) $n - 12 = 26$

(f) $12 - n = 26$

(g) $26 - 21 = n$

(h) $21 - 26 = n$

7. **BRAINTWISTER.**

(a) The two numbers you cperate on are n and n. The operation you use is addition. The result is 200. What number is n?

(b) Follow the directions of exercise (a) but replace 200 with 582.

8. **BRAINTWISTER.** What is wrong with this problem? The two numbers you operate on are n and n. The operation you use is subtraction. The result is 10. What number is n?

9. **BRAINTWISTER.** Two numbers operated on are n and 376. The result is 593. Write two true mathematical sentences using n, 376, and 593. In each mathematical sentence n will be a different number.

10. **BRAINTWISTER.** Two numbers operated on are n and 376. The result is 89. Can you write one or two true mathematical sentences using n, 376, and 89? Why?

## Exercise Set 18

1. A model plane costs $2.15. Joe had some money and then he earned $1.58. Then he had exactly enough to buy the plane. How much did he have before earning $1.58?

2. The fourth grade class collected 287 more pounds of old newspapers than the fifth grade class. The fourth grade class collected 512 pounds. How much did the fifth grade collect?

3. 721 is the largest 3 digit number that can be written using each of the digits 7, 2, and 1. What is the smallest number that can be so written? What must be added to the smaller number to get the larger?

4. Mary went to the store to buy one loaf of bread and one dozen eggs. Bread is 29¢ a loaf and eggs 65¢ a dozen. Using only the above information which of these questions can you answer?
   (a) What is the cost of Mary's purchases?
   (b) How much in all did Mary pay for bread?
   (c) How much change did she bring home?
   (d) If she gave the clerk a $5 bill, how much change did she receive?

5. The East School had a newspaper and magazine drive. Room A collected 1,546 pounds, Room B collected 2,875 pounds, and Room C collected 5,324 pounds. How many pounds of paper did these three rooms collect in all?

6. BRAINTWISTER. Use the numbers 2, 3, 4, 5, 6, 7, 8, 9 and 10 to make a magic square. Hint: the sum of each row, column, and diagonal is 18.

7. BRAINTWISTER. (a) What number is n if

$$(6 - n) + 4 = (6 + n) - 4$$

(b) How many counting numbers are there between 194 and 275?

8. BRAINTWISTER. Each mathematical sentence below is true. In which is n not a whole number?

(a) $n - n = n$

(b) $10 - n = n$

(c) $(3 + 2) + 2 = n$

(d) $(3 + 2) + n = 2$

9. BRAINTWISTER. Find n so each mathematical sentence is true.

(a) n is less than 2.

(b) n is less than 8 and n is more than 6.

(c) n added to 3 is less than 5.

(d) $n < 12$ and $n > 10$

(e) $n + 4 < 6$

## Exercise Set 19

At Jordon school, the cafeteria served lunch to:

195 children on Monday
218 children on Tuesday
198 children on Wednesday
203 children on Thursday
194 children on Friday

Use the above information to solve problems 1-8.

1. How many children were served lunch during the week?

2. How many more than 1,000 lunches were served during the week?

3. Find the two days on which the most lunches were served. The total number of lunches for these two days was how many less than 500?

4. The total number of lunches served the first three days of the week is how many more than the number served the last two days of the week?

The mathematical sentences in exercises 5 through 8 answer what questions about the number of lunches served?

5. $195 + n = 218$
6. $n = 198 + 194$
7. $(198 + 203) + 194 = n$
8. $218 - 203 = n$

The prices of some card games are: Old Maid 26¢, Hearts 19¢, Play Your Hunch 17¢, and Rummy 24¢.

Which of the mathematical sentences in the box can be used to answer exercises 9 through 11?

(a) $100 + n = 17 + 19$
(b) $(19 + 24) - 26 = n$
(c) $(24 + 19) + 26 = n$
(d) $n - (19 + 24) = 26$
(e) $100 - (17 + 19) = n$
(f) $(24 + 19) - n = 26$

9. What is the total cost of Rummy, Hearts, and Old Maid?

10. How much change do you receive from $1.00 if you buy Play Your Hunch and Hearts?

11. How much more will it cost to buy the 2 games Hearts and Rummy than 1 game of Old Maid?

The mathematical sentences in exercises 12 through 17 answer what questions about the cost of the card games?

12. $17 + 24 = n$

13. $n = 19 - 17$

14. $24 - n = 17$

15. $n + 26 = 19$

16. $n = 17 + 26$

17. $17 + n = 24$

## Exercise Set 20

### Meaning of Operation

You have been studying addition and subtraction, two of the operations of mathematics. They are operations on two numbers. The symbols that indicate these operations are + and -. Now we are going to "make up" some operations. They are "make-believe" operations and are not found in mathematics books. They have been invented to see if you can discover their meaning.

1. One make-believe operation is named "circ." The symbol to indicate circ is $\odot$ . $2 \odot 4$ is read, "Two circ four." Circ means add 3 to the first number and then subtract the second number from that sum. Thus $2 \odot 4 = 1$. Find n for each of these.

   (a) $3 \odot 2 = n$ (c) $5 \odot 1 = n$ (e) $7 \odot 6 = n$

   (b) $8 \odot 1 = n$ (d) $4 \odot 4 = n$ (f) $9 \odot 2 = n$

2. Another make-believe operation is named, "bow." The symbol to indicate bow is T . 3 T 4 is read, "Three bow four." Bow means choose the smaller number. Thus 8 T 5 = 5. Find n for each of these.

   (a) 2 T 3 = n (c) 4 T 1 = n (e) 8 T 3 = n

   (b) 12 T 8 = n (d) 9 T 10 = n (f) 7 T 6 = n

3. Another operation is named, "wob." The symbol to indicate wob is $\perp$. $3 \perp 4$ is read "Three wob four." Here are some results of the operation, wob, on two numbers. Try to find the meaning of wob.

(a) $4 \perp 6 = 6$

(b) $8 \perp 0 = 8$

(c) $8 \perp 1 = 8$

(d) $2 \perp 6 = 6$

(e) $5 \perp 9 = 9$

(f) $7 \perp 7 = 7$

Find n for each of the following:

(g) $5 \perp 6 = n$

(h) $1 \perp 9 = n$

(i) $7 \perp 10 = n$

(j) $6 \perp 8 = n$

(k) $2 \perp 0 = n$

(l) $9 \perp 2 = n$

4. The symbol * is to be a sign of operation. $3 * 4$ tells you to operate on 3 and 4 in a certain way. It is read, "Three star four." Here are some results of the operation, star, on two numbers. Try to find the meaning of star.

(a) $3 * 4 = 8$

(b) $5 * 6 = 12$

(c) $2 * 6 = 9$

(d) $3 * 7 = 11$

(e) $1 * 1 = 3$

(f) $5 * 4 = 10$

Find n for each of the following:

(g) $2 * 4 = n$

(h) $8 * 7 = n$

(i) $3 * 6 = n$

(j) $5 * 9 = n$

(k) $1 * 0 = n$

(l) $1 * 6 = n$

5. Another operation is called, "pick." The symbol for pick is ⌋. Try to find the meaning of ⌋ from these examples.

(a) 3 ⌋ 5 = 4 (c) 2 ⌋ 4 = 3 (e) 7 ⌋ 5 = 6

(b) 0 ⌋ 2 = 1 (d) 8 ⌋ 6 = 7 (f) 9 ⌋ 7 = 8

6. Another operation is called, "alpha." The symbol for alpha is ℒ. It is an operation on one number. Try to find the meaning of ℒ from these examples.

(a) ℒ 3 = 6 (b) ℒ 0 = 0 (c) ℒ 5 = 10 (d) ℒ 8 = 16

What is n in each of the following?

(e) ℒ 4 = n (f) ℒ 9 = n (g) ℒ 1 = n (h) ℒ 7 = n

7. SUPER BRAINTWISTER. Another operation is called, "beta." The symbol to indicate beta is **B**. Try to find the meaning of beta from these examples:

(a) 3 **B** 4 = 5 (c) 2 **B** 8 = 2 (e) 6 **B** 1 = 5

(b) 1 **B** 2 = 9 (d) 7 **B** 5 = 0 (f) 3 **B** 3 = 6

Find n for each of the following:

(g) 2 **B** 3 = n (i) 5 **B** 6 = n (k) 1 **B** 0 = n

(h) 8 **B** 4 = n (j) 4 **B** 2 = n (l) 5 **B** 5 = n

8. SUPER BRAINTWISTER. For which of the operations in exercises 1-7 does the commutative property seem to hold?

## Exercise Set 21

### UNION OF SETS

Pretend you have Set A and Set B.

Call Set C the intersection of Set A and Set B.

Call Set D the union of Set A and Set B.

Copy and fill in this table. (You may need to draw some pictures.)

| Number of members in Set A | Number of members in Set B | Number of members in Set C (Intersection) | Number of members in Set D (Union) |
|---|---|---|---|
| (1) 7 | 8 | 0 | 15 |
| (2) 7 | 8 | 2 | 13 |
| (3) 7 | 9 | 4 | ___ |
| (4) 8 | 3 | 0 | ___ |
| (5) 8 | 3 | n | ___ |
| (6) 8 | m | 0 | ___ |
| (7) p | r | 2 | ___ |
| (8) p | r | 5 | ___ |

## Chapter 7

# TECHNIQUES OF MULTIPLICATION AND DIVISION

### OPERATIONS

We think of addition, subtraction, multiplication, and division as the four basic operations of arithmetic.

We have learned that an operation on numbers is a way of thinking about two numbers and getting one number as a result.

When we think about 12 and 3 and get 15, we are *adding*. When we think about 12 and 3 and get 9, we are *subtracting*. When we think about 12 and 3 and get 36, we are *multiplying*. When we think about 12 and 3 and get 4, we are *dividing*.

## MULTIPLICATION

We express multiplication like this:

$$9 \times 4 = 36.$$

We read the sentence like this:

9 times 4 is equal to 36.
9 times 4 equals 36.

We know that:

9 is a factor of 36.
4 is a factor of 36.
36 is the product of 9 and 4.

## DIVISION

We express division like this:

$$36 \div 9 = n$$

$$36 = n \times 9$$

or

$$36 = 9 \times n.$$

We read the sentence like this:

36 divided by 9 is equal to n.
36 is equal to what times 9 ?
36 is equal to 9 times what number?

We know that:

36 is the product of 9 and n.
9 is a known factor of 36.
n is an unknown factor of 36.

## THE DISTRIBUTIVE PROPERTY OF MULTIPLICATION OVER ADDITION

Methods of multiplication depend on expressing one factor as a sum and then using the distributive property of multiplication over addition.

To multiply $48 \times 6$, you can think of 48 as $(40 + 8)$. Then we multiply each number by the factor 6. We use the distributive property.

| | |
|---|---|
| $6 \times 48 = 6 \times (40 + 8)$ | Rename 48 as $(40 + 8)$. |
| $= (6 \times 40) + (6 \times 8)$ | Distribute the 6 over $(40 + 8)$. |
| $= 240 + 48$ | The product of 6 and 40 is 240. The product of 6 and 8 is 48. |
| $= 288$ | The sum of 240 and 48 is 288. |

## THE DISTRIBUTIVE PROPERTY OF DIVISION OVER ADDITION

To divide 75 by 5, we express 75 as $(50 + 25)$. Then we divide both numbers by 5.

| | |
|---|---|
| $75 \div 5 = (50 + 25) \div 5$ | Rename 75 as $(50 + 25)$. |
| $= (50 \div 5) + (25 \div 5)$ | Distribute the 5 over $(50 + 25)$. |
| $= 10 + 5$ | Divide 50 by 5. Divide 25 by 5. |
| $= 15$ | The sum of 10 and 5 is 15. |

## Exercise Set 1

Write the numerals from 1 to 20 on your paper. If a statement is true, write true. If a statement is false, write false.

1. Using the set of whole numbers, you can multiply any pair of numbers and always get a whole number for their product. ____

2. Using the set of whole numbers, you can divide any pair of numbers and always get a whole number for the unknown factor. ______

3. $273 \times 846 = 846 \times 273$
4. $3 + 1 = 1$
5. $69 + 3 = 3 + 69$
6. $17 = 17 + 1$
7. $6 \times 0 = 6$
8. $1 \times 9 = 9$
9. $0 + 6 = 0$
10. $6 \times 9 < 7 \times 9$
11. $58 \times 69 > 69 \times 58$
12. $48 + 4 = (40 + 4) + (8 + 4)$
13. $(20 + 4) \times 7 = (20 \times 7) + (4 \times 7)$
14. $2 \times (3 \times 17) = (3 \times 17) \times 2$
15. $2 \times 34 = (2 \times 30) + (2 \times 40)$
16. $(21 \times 7) + 7 = (21 + 7) \times 7$
17. $(48 + 6) + 2 = 48 + (6 + 2)$
18. $(5 \times 30) + (5 \times 6) = 5 \times 36$
19. $(47 \times 18) + (47 \times 12) = 47 \times 30$
20. $(12 + 3) + (12 + 3) = 24 + 3$

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## Exercise Set 2

Find a decimal numeral for n in each sentence.

1. 10 × 18 = n
2. 17 × 10 = n
3. 27 × 10 = n
4. 10 × 35 = n
5. 10 × 107 = n
6. 12 × 10 = n
7. 120 × 10 = n
8. 10 × 19 = n
9. 30 × 100 = n
10. 300 × 10 = n
11. 10 × 47 = n
12. 89 × 10 = n
13. 54 × 10 = n
14. 10 × 98 = n
15. 10 × 125 = n
16. 314 × 10 = n
17. 412 × 10 = n
18. 842 × 10 = n

BRAINTWISTERS:

19. 17 × 20 = n
20. 12 × 30 = n

## MULTIPLYING BY MULTIPLES OF TEN

We have learned how to multiply two numbers when one of the numbers is 10.

Now we want to learn to multiply two numbers when one of the numbers is a multiple of 10. Multiples of 10 are 10, 20, 30, 40, 50, and so on. Can you name some other multiples of 10?

Suppose we find the product of 7 and 20. To multiply 7 and 20, we can think of 20 as $(10 + 10)$. Then,

| | |
|---|---|
| $7 \times 20 = 7 \times (10 + 10)$ | Rename 20 as $(10 + 10)$. |
| $= (7 \times 10) + (7 \times 10)$ | Distribute 7 over $(10 + 10)$. |
| $= 70 + 70$ | Multiply 7 and 10. |
| $= 140$ | Add 70 and 70. |

Too, we can think of 20 as $(2 \times 10)$. Then,

| | |
|---|---|
| $7 \times 20 = 7 \times (2 \times 10)$ | Rename 20 as $(2 \times 10)$. |
| $= (7 \times 2) \times 10$ | Use the associative property. |
| $= 14 \times 10$ | Multiply 7 and 2. |
| $= 140$ | Multiply 14 and 10. |

Is it easier to find the product of 7 and 20 by the first way or the second way? Let us find the product of another pair of numbers using the second way. One of the factors is a multiple of 10. Give reasons for each step in the following example.

$$\begin{aligned} 8 \times 40 &= 8 \times (4 \times 10) \\ &= (8 \times 4) \times 10 \\ &= 32 \times 10 \\ &= 320 \end{aligned}$$

The product of 8 and 40 is 320.

$$8 \times 40 = 320.$$

## Exercise Set 3

Find the decimal numeral for each product in exercises 1 through 12. In exercises 1 through 4, write each step as in the example.

Example: $7 \times 30 = 7 \times (3 \times 10)$
$= (7 \times 3) \times 10$
$= 21 \times 10$
$= 210$

1. $5 \times 80$

2. $70 \times 7$

3. $50 \times 8$

4. $6 \times 30$

In exercises 5 through 12, find the decimal numeral for each product without writing all the steps. In exercise 5, can you think that 7 multiplied by 4 is 28 and that 28 multiplied by 10 is 280?

5. $7 \times 40$
6. $5 \times 60$
7. $6 \times 70$
8. $7 \times 80$
9. $60 \times 9$
10. $3 \times 600$
11. $30 \times 8$
12. $9 \times 90$

## Exercise Set 4

Use mathematical sentences to help solve each of the following problems. Express each answer in a complete sentence.

1. The beads on an abacus may be arranged so that there are 20 beads on each of 4 wires. How many beads are on this abacus?

2. Some land will be divided into 7 blocks. 40 houses will be built on each block. How many houses will there be on the land?

3. In one section of a plane there were 20 rows of seats with 5 seats in each row. How many seats were there in this section of the plane?

4. At an assembly the chairs were arranged in 30 rows. There were 10 chairs in each row. How many chairs were set up for the assembly?

5. Bob bought 3 season tickets to the basketball games. Each ticket costs $3.20. How much did Bob spend for the tickets?

6. On the family room floor there were 660 tiles. 304 tiles were used on the kitchen floor. How many more tiles were used on the floor of the family room than on the floor of the kitchen?

# MULTIPLYING BY MULTIPLES OF ONE HUNDRED

We have learned how to multiply two whole numbers when one of the numbers is 10. We have learned how to multiply two whole numbers when one is a multiple of 10. What are some multiples of 10?

Look at this example for finding the product of 6 and a multiple of 10 (30).

$$\begin{aligned} 6 \times 30 &= 6 \times (3 \times 10) \\ &= (6 \times 3) \times 10 \\ &= 18 \times 10 \\ &= 180 \end{aligned}$$

We now want to learn to multiply two whole numbers when one of the numbers is 100. We also want to learn how to find the product of two numbers when one factor is a multiple of 100. What are multiples of 100? Is 200 a multiple of 100? is 300? is 400? Can you name some other multiples of 100?

See if you can understand these examples.

Example 1:
$$\begin{aligned} 6 \times 100 &= 6 \times (10 \times 10) \\ &= (6 \times 10) \times 10 \\ &= 60 \times 10 \\ &= 600 \end{aligned}$$

Example 2:
$$\begin{aligned} 18 \times 100 &= 18 \times (10 \times 10) \\ &= (18 \times 10) \times 10 \\ &= 180 \times 10 \\ &= 1800 \end{aligned}$$

Example 3:
$$\begin{aligned} 6 \times 300 &= 6 \times (3 \times 100) \\ &= (6 \times 3) \times 100 \\ &= 18 \times 100 \\ &= 1800 \end{aligned}$$

Example 4:
$$\begin{aligned} 50 \times 30 &= (5 \times 10) \times (3 \times 10) \\ &= (5 \times 3) \times (10 \times 10) \\ &= 15 \times 100 \\ &= 1500 \end{aligned}$$

Example 5:
$$\begin{aligned} 16 \times 200 &= 16 \times (2 \times 100) \\ &= (16 \times 2) \times 100 \\ &= 32 \times 100 \\ &= 3200 \end{aligned}$$

Example 6:
$$\begin{aligned} 4 \times 2000 &= 4 \times (20 \times 100) \\ &= (4 \times 20) \times 100 \\ &= 80 \times 100 \\ &= 8000 \end{aligned}$$

Can you name the product just by looking at the two numbers to be multiplied? Try these.

$87 \times 10$

$5 \times 60$

$40 \times 30$

$4 \times 100$

$200 \times 3$

$12 \times 400$

How many could you do?

Now you can use what you have learned about multiplying by 10 and 100 and their multiples.

## Exercise Set 5

Copy and complete each of the following.

1. 7 × 10 = ____

2. 5 × 500 = ____

3. ____ = 3 × 600

4. 100 × 8 = ____

5. ____ = 9 × 10

6. 10 × 7 = ____

7. 500 × 5 = ____

8. 600 × 3 = ____

9. 8 × 60 = ____

10. 50 × 6 = ____

11. ____ = 500 × 6

12. ____ = 7 × 800

13. 400 × 3 = ____

14. 7 × 500 = ____

15. 800 × 5 = ____

16. 50 × 60 = ____

17. 60 × 90 = ____

18. 400 × 20 = ____

19. 3 × 2000 = ____

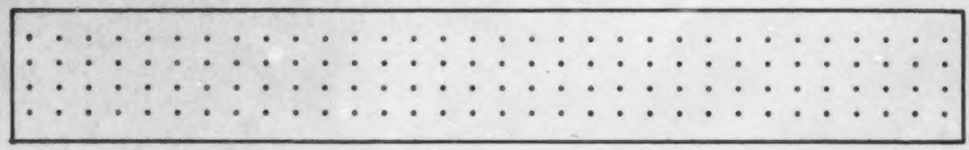
20. 6 × 3000 = ____

## MORE ABOUT MULTIPLYING

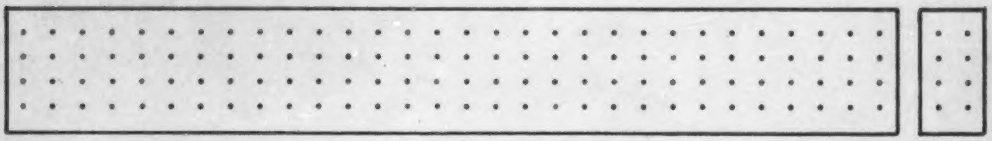
We now learn to multiply two numbers like these:

4 and 32

This array helps us to think about 4 × 32.



We can make smaller arrays.



How many rows does each have? How many columns does each have? We write

$$\begin{aligned} 4 \times 32 &= 4 \times (30 + 2) \\ &= (4 \times 30) + (4 \times 2) \\ &= 120 + 8 \\ &= 128 \end{aligned}$$

We can use one of these ways too.

$$\begin{array}{r} 32 \\ \underline{\times\ 4} \end{array} \quad \text{or} \quad \begin{array}{c} 30 + 2 \\ \underline{\qquad \times\ 4} \\ 120 + 8 = 128 \end{array} \quad \text{or} \quad \begin{array}{rl} 32 & \\ \underline{\times\ 4} & \\ 120 & \longleftarrow (4 \times 30) \\ \underline{\phantom{1}8} & \longleftarrow (4 \times 2) \\ 128 & \end{array} \quad \begin{array}{r} 32 \\ \underline{\times\ 4} \\ 8 \\ \underline{120} \\ 128 \end{array}$$

Can you think of another way?

## Exercise Set 6

Find the decimal numeral for each product in the following exercises. Use two forms as in the example.

Example:

$$\begin{aligned} 3 \times 12 &= 3 \times (10 + 2) \\ &= (3 \times 10) + (3 \times 2) \\ &= 30 + 6 \\ &= 36 \end{aligned} \qquad \begin{array}{r} 12 \\ \underline{\times 3} \\ 6 \\ \underline{30} \\ 36 \end{array}$$

1. $3 \times 32$ $\qquad \begin{array}{r} 32 \\ \underline{\times 3} \end{array}$

2. $4 \times 23$ $\qquad \begin{array}{r} 23 \\ \underline{\times 4} \end{array}$

3. $6 \times 34$ $\qquad \begin{array}{r} 34 \\ \underline{\times 6} \end{array}$

4. $4 \times 65$ $\qquad \begin{array}{r} 65 \\ \underline{\times 4} \end{array}$

5. $4 \times 82$ $\qquad \begin{array}{r} 82 \\ \underline{\times 4} \end{array}$

## Exercise Set 6 (Cont'd)

6. 5 × 87 $\begin{array}{r} 87 \\ \underline{\times\ 5} \end{array}$

7. 7 × 34 $\begin{array}{r} 34 \\ \underline{\times\ 7} \end{array}$

8. 8 × 37 $\begin{array}{r} 37 \\ \underline{\times\ 8} \end{array}$

9. 4 × 36 $\begin{array}{r} 36 \\ \underline{\times\ 4} \end{array}$

10. 8 × 89 $\begin{array}{r} 89 \\ \underline{\times\ 8} \end{array}$

## MULTIPLYING LARGER NUMBERS

We know how to find the products of numbers like 3 and 46, 7 and 39, 6 and 45. What number must n represent in each of these sentences if the sentence is a true statement?

$$3 \times 46 = n$$

$$7 \times 39 = n$$

$$6 \times 45 = n$$

The products are 273, 138, and 270. Now match the products and the product expressions.

We now want to find the product of numbers like 3 and 312.

We write

$$\begin{aligned} 3 \times 312 &= 3 \times (300 + 10 + 2) \\ &= (3 \times 300) + (3 \times 10) + (3 \times 2) \\ &= 900 + 30 + 6 \\ &= 936 \end{aligned}$$

There are several ways that we might use the vertical form for multiplication. Here are some of them.

$$\begin{array}{r} 300 + 10 + 2 \\ \times 3 \\ \hline 900 + 30 + 6 = 936 \end{array}$$

*or*

$$\begin{array}{rl} 312 & \\ \times 3 & \\ \hline 900 & \longleftarrow (3 \times 300) \\ 30 & \longleftarrow (3 \times 10) \\ 6 & \longleftarrow (3 \times 2) \\ \hline 936 & \longleftarrow (3 \times 312) \end{array}$$

*or*

$$\begin{array}{r} 312 \\ \times 3 \\ \hline 6 \\ 30 \\ 900 \\ \hline 936 \end{array}$$

In the last example, how did we get 900, 30, and 6?

You do not need to use all of these ways. Use the one that you like best. You may even like a short form like this.

$$\begin{array}{r} 312 \\ \times 3 \\ \hline 936 \end{array}$$

Can you discover a way to find the product of 4 and 2102?

## Exercise Set 7

Find the decimal numeral for each product in the following exercises. Show the partial products.

1. 2 × 311
2. 2 × 434
3. 4 × 322
4. 3 × 412
5. 3 × 210
6. 2 × 303
7. 4 × 300
8. 5 × 601
9. 8 × 711
10. 3 × 3020
11. 4 × 3002
12. 7 × 5101

## Exercise Set 8

Work these exercises as in the example. Use the vertical form if you can.

Example: $\begin{array}{r} 311 \\ \underline{\times\ 5} \\ 1{,}555 \end{array}$

1. $\begin{array}{r} 243 \\ \underline{\times\ 2} \end{array}$
2. $\begin{array}{r} 210 \\ \underline{\times\ 4} \end{array}$
3. $\begin{array}{r} 203 \\ \underline{\times\ 3} \end{array}$
4. $\begin{array}{r} 202 \\ \underline{\times\ 4} \end{array}$
5. $\begin{array}{r} 420 \\ \underline{\times\ 2} \end{array}$
6. $\begin{array}{r} 800 \\ \underline{\times\ 6} \end{array}$
7. $\begin{array}{r} 821 \\ \underline{\times\ 4} \end{array}$
8. $\begin{array}{r} 3020 \\ \underline{\times\ 3} \end{array}$
9. $\begin{array}{r} 3002 \\ \underline{\times\ 4} \end{array}$
10. $\begin{array}{r} 502 \\ \underline{\times\ 3} \end{array}$
11. $\begin{array}{r} 134 \\ \underline{\times\ 2} \end{array}$
12. $\begin{array}{r} 612 \\ \underline{\times\ 4} \end{array}$
13. $\begin{array}{r} 723 \\ \underline{\times\ 3} \end{array}$
14. $\begin{array}{r} 632 \\ \underline{\times\ 3} \end{array}$
15. $\begin{array}{r} 734 \\ \underline{\times\ 2} \end{array}$
16. $\begin{array}{r} 1010 \\ \underline{\times\ 9} \end{array}$
17. $\begin{array}{r} 1023 \\ \underline{\times\ 3} \end{array}$
18. $\begin{array}{r} 2332 \\ \underline{\times\ 3} \end{array}$
19. $\begin{array}{r} 8212 \\ \underline{\times\ 4} \end{array}$
20. $\begin{array}{r} 9111 \\ \underline{\times\ 8} \end{array}$

# A SHORTER METHOD OF MULTIPLYING

In our last lesson you saw how to shorten the form at the left so that you could multiply 312 by 3 in the way shown at the right:

$$\begin{array}{r} 312 \\ \underline{\times\ \ 3} \\ 6 \\ 30 \\ \underline{900} \\ 936 \end{array} \qquad\qquad \begin{array}{r} 312 \\ \underline{\times\ \ 3} \\ 936 \end{array}$$

Were you able to tell what you had to think to use this way? We want to think some more about these shorter ways.

Suppose we need to find the product of 4 and 23. We already know how to find the product in this way:

$$\begin{array}{r} 23 \\ \underline{\times\ 4} \\ 12 \\ \underline{80} \\ 92 \end{array}$$

Now let us see if we can find a shorter form for doing this.

$$\begin{array}{r} 23 \\ \underline{\times\ \ 4} \\ 2 \end{array}$$

We can think 4 × 3 = 12. Think of 12 as 1 ten and 2 ones. Let us write the 2 in the one's place. We will remember the 1 ten.

$$\begin{array}{r} 23 \\ \underline{\times\ \ 4} \\ 92 \end{array}$$

4 × 20 = 80. 80 is 8 tens. 8 tens and 1 ten are 9 tens. How can we write the numeral for 9 tens and 2 ones?

What is the product of 4 and 23? We can write

4 × 23 = 92.

Here are some problems. Can you fill in the blanks?

$$\begin{array}{r} 45 \\ \underline{\times\ 3} \\ 13\_\_ \end{array} \qquad\qquad \begin{array}{r} 82 \\ \underline{\times\ 6} \\ 4\_\_2 \end{array}$$

$$\begin{array}{r} 37 \\ \underline{\times\ 5} \\ \_\_5 \end{array} \qquad\qquad \begin{array}{r} 74 \\ \underline{\times\ 9} \\ 66\_\_ \end{array}$$

$$\begin{array}{r} 28 \\ \underline{\times\ 3} \\ \_\_4 \end{array}$$

You may wish to use this short method to do the problems in Exercise Set 9. You may wish to try several methods.

## Exercise Set 9

Use the way you like best to find the number represented by n in each of these.

1. 4 × 17 = n
2. 7 × 33 = n
3. 9 × 62 = n
4. 6 × 81 = n
5. 7 × 87 = n
6. 9 × 56 = n
7. 5 × 52 = n
8. 3 × 89 = n
9. 6 × 56 = n
10. 8 × 78 = n

Copy and complete. (Note: Each blank must be replaced by one digit.)

11.
__5
× 5
225

12.
3__
× 6
228

13.
__9
× 7
413

14.
__9
× 5
395

15.
__8
× 4
_ _

16.
__5
× 8
3_ _

17.
1_ _
× 6
6_ _

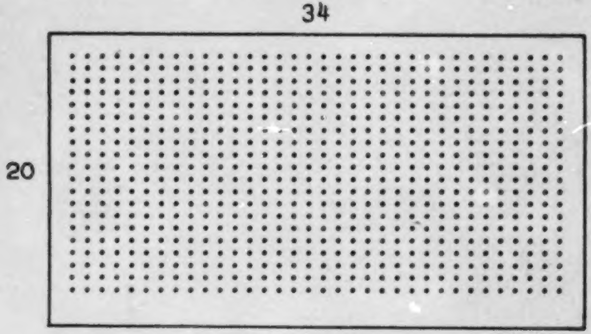
## MULTIPLYING NUMBERS LESS THAN 100 BY MULTIPLES OF 10

We now want to learn how to multiply numbers like 20 and 34, 40 and 46, 50 and 23, 60 and 31.

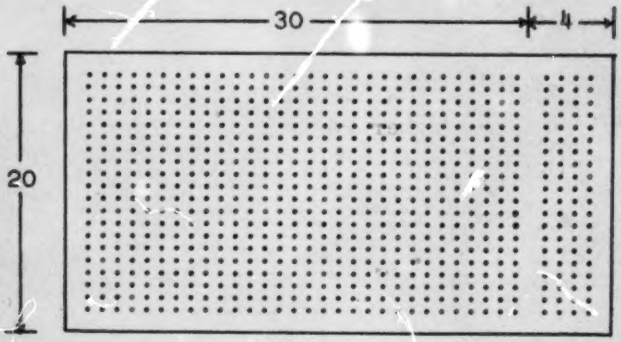
Is one number of each pair a multiple of 10? Name the multiples of 10 in these pairs.

Is the other number in each pair less than 100?

Let us learn a way to find the product of 20 and 34. Does 20 × 34 name the number of dots in this array? How can you tell?



Here is another array just like the one above. Let us separate it into smaller arrays.



Does $(20 \times 34)$ describe the first array?

Does $(20 \times 30) + (20 \times 4)$ describe the second array?

Do the two arrays show that $20 \times 34 = (20 \times 30) + (20 \times 4)$?

Does this help you to find the number of dots in the big array?

We can write:

$$\begin{aligned} 20 \times 34 &= 20 \times (30 + 4) \\ &= (20 \times 30) + (20 \times 4) \end{aligned}$$

$$\begin{array}{l} 30 \ + \ \ 4 \\ \underline{\qquad\quad \times\ 20} \\ 600 \ + \ 80 = 680 \end{array} \qquad\qquad \begin{array}{rl} 34 & \\ \underline{\times\ 20} & \\ 80 & (20 \times 4) \\ \underline{600} & \underline{(20 \times 30)} \\ 680 & (20 \times 34) \end{array}$$

Can you think of other ways of finding the product of these two numbers?

## Exercise Set 10

Find the number n represents for each of these.

1. 30 × 30 = n
2. 20 × 40 = n
3. 10 × 80 = n
4. 30 × 40 = n
5. 20 × 50 = n
6. 60 × 90 = n
7. 70 × 70 = n
8. 80 × 40 = n
9. 30 × 90 = n
10. 50 × 70 = n
11. 20 × 60 = n
12. 70 × 80 = n
13. 20 × 43 = n
14. 30 × 33 = n
15. 10 × 87 = n
16. 50 × 32 = n
17. 50 × 62 = n
18. 70 × 83 = n
19. 90 × 65 = n
20. 80 × 87 = n
21. 90 × 38 = n
22. 70 × 57 = n
23. 40 × 93 = n
24. 60 × 83 = n

## FINDING PRODUCTS OF NUMBERS GREATER THAN 10 (AND LESS THAN 100)

We have learned to find the product of pairs of numbers. We made some choices in the numbers we selected.

We learned to find products of pairs of numbers like these:

3 and 45

$$\begin{aligned} 3 \times 45 &= 3 \times (40 + 5) \\ &= (3 \times 40) + (3 \times 5) \\ &= 120 + 15 \\ &= 135 \end{aligned}$$

$$\begin{array}{r l} 45 & \\ \underline{\times 3} & \\ 120 & (3 \times 40) \\ \underline{15} & (3 \times 5) \\ 135 & \end{array}$$

8 and 16

$$\begin{aligned} 8 \times 16 &= 8 \times (10 + 6) \\ &= (8 \times 10) + (8 \times 6) \\ &= 80 + 48 \\ &= 128 \end{aligned}$$

$$\begin{array}{r l} 16 & \\ \underline{\times 8} & \\ 128 & (8 \times 6) + (8 \times 10) \end{array}$$

5 and 24

$$\begin{aligned} 5 \times 24 &= 5 \times (20 + 4) \\ &= (5 \times 20) + (5 \times 4) \\ &= 100 + 20 \\ &= 120 \end{aligned}$$

$$\begin{array}{r l} 24 & \\ \underline{\times 5} & \\ 120 & (5 \times 4) + (5 \times 20) \end{array}$$

We can use the same way for all of these examples. We can use different ways to find the product of numbers like these.

We learned to find products of numbers when one of the numbers was a multiple of 10. What numbers less than 100 are multiples of 10?

Can you find the product for each of these pairs?

20 and 45

$$\begin{array}{r} 45 \\ \underline{\times\ 20} \end{array}$$

10 and 17

$$\begin{array}{r} 17 \\ \underline{\times\ 10} \end{array}$$

37 and 20

$$\begin{array}{r} 20 \\ \underline{\times\ 37} \end{array} \quad \text{or} \quad \begin{array}{r} 37 \\ \underline{\times\ 20} \end{array}$$

Why can we change order?

The products are 900, 170, and 740. Did you get them right?

Now we will learn how to find the products of any two numbers greater than 10. We will still make a choice. They will be numbers less than 100.

Ann was cutting a large cake.

She cut 12 long pieces (rows) of cake.

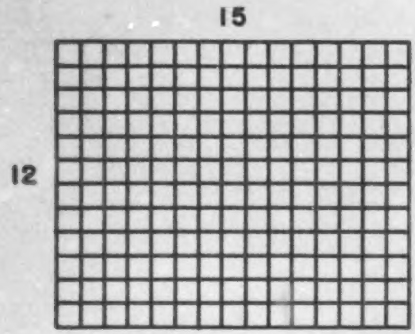
She cut each long piece into 15 pieces.

How many pieces of cake did she have?

Is this mathematical sentence a correct one for this problem?

$12 \times 15 = n$ Why?

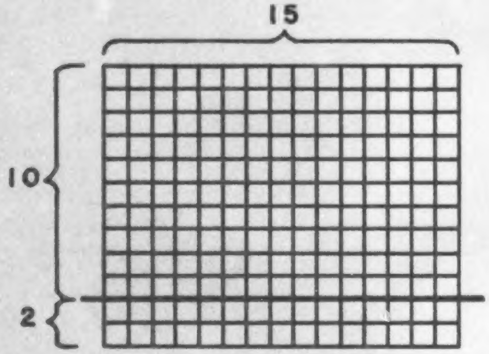
Does this picture give us an idea of the cake? Explain.



Let us separate this array into smaller arrays.

What product expression does each array suggest to you?

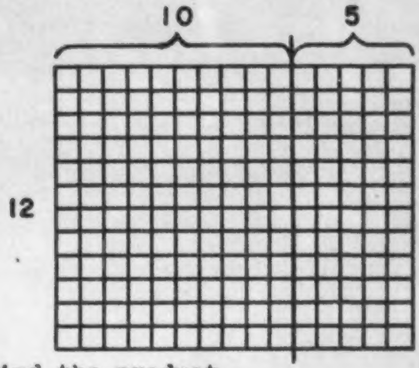
Find the array each of these describes.



$$\begin{array}{r} 10 \times 15 = 150 \\ \underline{2 \times 15 = \phantom{0}30} \\ 12 \times 15 = 180 \end{array}$$

There are other ways we can separate the array. Look at this one.

$$\begin{aligned} 12 \times 15 &= 12 \times (10 + 5) \\ &= (12 \times 10) + (12 \times 5) \\ &= 120 + 60 \\ &= 180 \end{aligned}$$

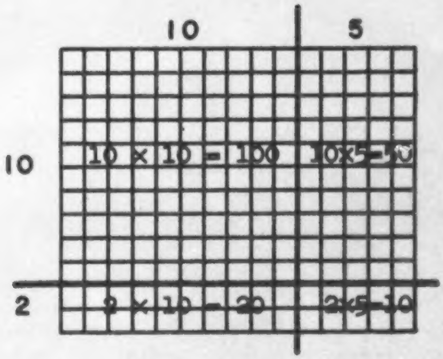


This vertical form helps us to find the product.

$$\begin{array}{rl} 15 & \\ \underline{\times\ 12} & \\ 120 & (12 \times 10) \\ \underline{60} & \underline{(12 \times 5)} \\ 180 & (12 \times 15) \end{array}$$

Let us see still another way of separating the array. Now we can write the product in vertical form in this way.

$$\begin{array}{rl} 15 & \\ \underline{\times\ 12} & \\ 10 & (2 \times 5) \\ 20 & (2 \times 10) \\ 50 & (10 \times 5) \\ \underline{100} & \underline{(10 \times 10)} \\ 180 & (12 \times 15) \end{array}$$



Use the way that you like best to find the products in the exercises in Exercise Set 11. (The more we can learn to remember, the less we need to write.)

## Exercise Set 11

Find the decimal numeral representing n.

1. 12 × 23 = n
2. 11 × 42 = n
3. 14 × 52 = n
4. 15 × 32 = n
5. 32 × 41 = n
6. 23 × 63 = n
7. 21 × 78 = n
8. 31 × 66 = n
9. 32 × 59 = n
10. 45 × 45 = n
11. 37 × 48 = n
12. 29 × 54 = n

(Optional)

Sometimes properties of multiplication can be used to make short cuts in multiplication. See if you can explain these short cuts.

13. 20 78
× 78 short cut: rewrite or rethink as × 20

14. 33 990
× 29 short cut: write or think as - 33
957

15. 101 7800
× 78 short cut: use short cut or think as + 78
7878

16. 480
× 370 short cut: find 37 × 48, then multiply by 100.

Try to find a short cut in these exercises. If you can't, do them in the usual way.

17. 50 19. 203
× 49 × 32

18. 30 20. 500
× 86 × 680

## USING MULTIPLICATION IN PROBLEM SOLVING

You have solved problems before.

Do you remember how you solved problems?

Let us use this problem to help us remember.

Problem: At the circus, the children of Madison School sat in a section of 15 rows. Eighteen children were seated in each row. How many children from Madison School were seated in this section?

Bits of Information: There are 15 rows and there are 18 children in each row.

Mathematical Sentence: $15 \times 18 = n$

Work:

$$\begin{array}{r} 18 \\ \times\ 15 \\ \hline 40 \\ 50 \\ 80 \\ 100 \\ \hline 270 \end{array}$$

Answer Sentence: 270 children were seated in this section.

In solving problems you need to:

Understand the question that is to be answered.

Find the information given in the problem that will help you.

Write a mathematical sentence that relates this information to the question.

Find the number that is not known.

Write an answer to the problem question.

## Exercise Set 12

1. The children of Madison School went to the circus in 6 buses. Forty-five children rode in each bus. How many children rode in the 6 buses?

2. There were 424 boys and girls enrolled in Madison School. If 270 children went to the circus and the other children went to the zoo, how many children went to the zoo?

3. One day at the zoo there were 154 children from Madison School and 168 children from Adams School. How many children visited the zoo that day from the two schools?

4. A crossword puzzle had 15 squares across and 12 squares down. How many squares were there in the puzzle?

5. There were 360 dots in one part of an array and 24 dots in the other part. How many dots were there in the whole array?

6. The score of a football game is 35 to 17. How many points does one team need to tie the score?

7. Mrs. Smith buys 14 gallons of milk each month. How many gallons does she buy in a year?

8. Fifteen gallons of ice cream were bought for the Halloween party. If one gallon served 26 children, how many children did the 15 gallons serve?

9. There were 12 tables in the cafeteria. If 16 children sat at each table, how many children could be served at one time?

10. 36 boxes of crayons were ordered for a class. Each box contained 24 crayons. How many crayons were there altogether?

11. In the parking lot at the ball park there were 24 rows with spaces for 35 cars in each row. How many cars may be parked in this lot?

12. It took 191 seconds for the children in Lowell School to leave the building during a fire drill in March. In April the time was 186 seconds. How much longer did it take the children to leave the building in March?

13. Mrs. Wood made 27 jars of jam. If each jar held 16 ounces, how many ounces did she make?

14. There are 53 boys and girls in the morning kindergarten class and 48 in the afternoon class. How many children are in the two classes?

15. Each of the 32 children in Miss Park's class made 18 name tags for open house. How many name tags did they make?

## FINDING UNKNOWN FACTORS

Try to find the unknown factors in these sentences. Use multiplication facts to help you.

| A | B |
|---|---|
| $4 \times n = 36$ | $4 \times n = 360$ |
| $n \times 8 = 64$ | $n \times 8 = 640$ |
| $n \times 7 = 42$ | $n \times 7 = 420$ |
| $2 \times n = 12$ | $2 \times n = 1200$ |
| $5 \times n = 30$ | $5 \times n = 3000$ |

What multiplication fact did you use for each?

In what way did finding the unknown factors in A help you to find the factors in B?

How can each of the sentences in A and B be rewritten using the division symbol?

Many problems are solved by dividing one number by another. Here is an example.

Paul has 52 stamps. He can put 4 stamps in one row in his book. How many rows will he need if he puts all 52 stamps in his book?

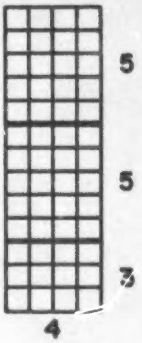
We want to find the number of rows of stamps.

There are 4 stamps in each row.

There are 52 stamps in all.

Is the mathematical sentence for this problem $n \times 4 = 52$, where $n$ represents the number of rows?

Think of an array.



The stamps in his book might be arranged as in the picture at the right. The way the array is separated shows that

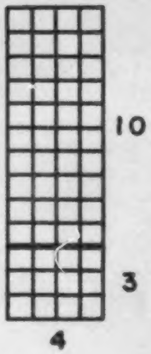
$$52 = 20 + 20 + 12.$$

We write:

$$\begin{aligned} 52 \div 4 &= (20 + 20 + 12) \div 4 \\ &= (20 \div 4) + (20 \div 4) + (12 \div 4) \\ &= 5 + 5 + 3 \\ &= 13. \end{aligned}$$

There are 13 rows of stamps.

The stamps in his book might also be arranged as in this picture at the right.



The way the array is separated shows that 52 = 40 + 12.

We write:

$$52 \div 4 = (40 + 12) \div 4$$
$$= (40 \div 4) + (12 \div 4)$$
$$= 10 + 3$$
$$= 13.$$

The number 52 can be renamed so that each addend is a multiple of 4. These numbers are multiples of 4:

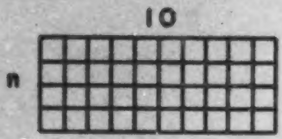
4, 8, 16. 20, ... .

Can you name some others?

Try some other ways of renaming 52.

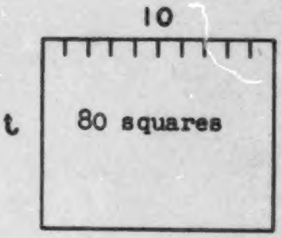
## Exercise Set 13

Find the unknown number in each of these exercises.

1. 

$n \times 10 = 40$

$n =$ ______

2. 

$t \times 10 = 80$

$t =$ ______

3. $1200 \div 3 = n$

4. $810 \div 9 = t$

5. $p \times 7 = 35$

6. $420 \div 6 = r$

7. $q = 640 \div 8$

8. $y = 770 \div 7$

## Exercise Set 14

Rename each product using multiples of the known factor as addends.

**Example 1:**

$n = 24 \div 2$
$= (20 + 4) \div 2$
$= (20 \div 2) + (4 \div 2)$
$= 10 + 2$
$= 12$

**Example 2:**

$r = 393 \div 3$
$= (300 + 90 + 3) \div 3$
$= (300 \div 3) + (90 \div 3) + (3 \div 3)$
$= 100 + 30 + 1$
$= 131$

1. $48 \div 4 = t$
2. $68 \div 2 = s$
3. $96 \div 3 = n$
4. $64 \div 2 = s$
5. $48 \div 2 = m$
6. $42 \div 2 = k$
7. $h = 88 \div 4$
8. $n = 55 \div 5$
9. $75 \div 5 = m$
10. $63 \div 3 = t$
11. $96 \div 6 = r$
12. $91 \div 7 = s$
13. $112 \div 8 = t$
14. $217 \div 7 = w$
15. $333 \div 9 = m$
16. $400 \div 5 = n$
17. $639 \div 3 = k$
18. $420 \div 4 = m$
19. $770 \div 7 = p$
20. $630 \div 6 = t$

## Exercise Set 15

Write mathematical sentences to help you solve the following problems. Solve them. Write an answer sentence for each problem.

1. At the air show 40 planes flew in formation. There were 10 rows. How many planes were in each row?

2. A class of 36 children was divided equally into 3 committees to plan a party. How many children were on each committee?

3. The grocer put 60 carrots in bunches of 5. How many bunches did he make?

4. How many days are there in 7 weeks?

5. Bob received an allowance of 50¢. Jim's allowance was 75¢. How much more money does Jim receive than Bob?

6. 50¢ is how many times as much money as 5¢?

7. The 80 men in the marching band were divided into 8 rows. How many men were in each row?

8. A jet plane carried 42 passengers in one section and 102 passengers in another section. How many passengers were aboard the jet?

## A WAY OF DIVIDING TWO NUMBERS

We have learned that we can rename the product and divide to find the unknown factor.

$$n = 225 \div 9$$

$= (18$[illegible]$+ 45) \div 9$

$$= (180 \div 9) + (45 \div 9)$$

$$= \quad 20 \quad + \quad 5$$

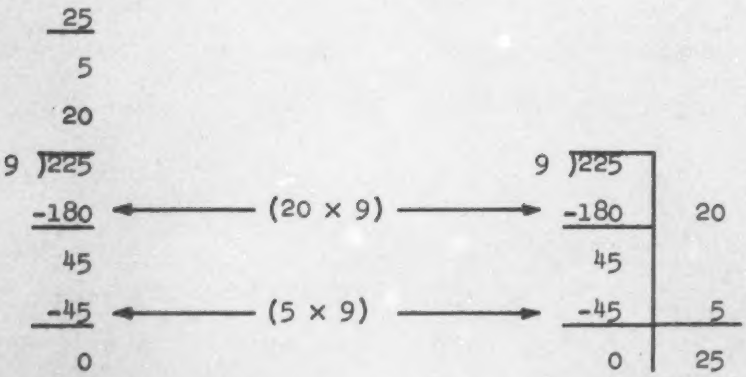
$$= 25$$

Here is another way to show division.

Mathematical Sentence:

$$n = 225 \div 9$$

Form I: Form II:


25
5
20
9 )225
-180
45
-45
0
(20 × 9)
(5 × 9)
9 )225
-180
45
-45
0
20
5
25

You may use either form shown above.

Now we know

$$\underline{25} \times 9 = 225 \qquad \text{or} \qquad 225 \div 9 = \underline{25}.$$

## Exercise Set 16

Use Form I or Form II to find the unknown number in each sentence.

1. $45 \div 3 = n$

2. $76 \div 4 = n$

3. $84 \div 3 = n$

4. $96 \div 8 = n$

5. $72 \div 3 = n$

6. $69 \div 3 = n$

7. $84 \div 7 = n$

8. $60 \div 12 = n$

9. $96 \div 3 = n$

10. $132 \div 6 = n$

11. $444 \div 6 = n$

12. $m = 207 \div 9$

13. $325 \div 5 = p$

14. $376 \div 8 = q$

## MORE ABOUT DIVIDING TWO NUMBERS

You have found that there are several ways to rename a product when you divide. Here are some ways that you may have used to find what number times 6 is 444. $n \times 6 = 444$. Using division we write: $444 \div 6 = n$. You may use either of these forms.

Form I.

(a)
$$\begin{array}{r}
\underline{74}\\
4\\
20\\
50\\
6\,\overline{)444}\\
\underline{300}\\
144\\
\underline{120}\\
24\\
\underline{24}\\
0
\end{array}$$

(b)
$$\begin{array}{r}
\underline{74}\\
4\\
10\\
60\\
6\,\overline{)444}\\
\underline{360}\\
84\\
\underline{60}\\
24\\
\underline{24}\\
0
\end{array}$$

(c)
$$\begin{array}{r}
\underline{74}\\
4\\
70\\
6\,\overline{)444}\\
\underline{420}\\
24\\
\underline{24}\\
0
\end{array}$$

Form II.

(a)
$$\begin{array}{r|l}
6\,\overline{)444} & \\
\underline{300} & 50\\
144 & \\
\underline{120} & 20\\
24 & \\
24 & 4\\ \hline
0 & 74
\end{array}$$

(b)
$$\begin{array}{r|l}
6\,\overline{)444} & \\
\underline{360} & 60\\
84 & \\
\underline{60} & 10\\
24 & \\
24 & 4\\ \hline
0 & 74
\end{array}$$

(c)
$$\begin{array}{r|l}
6\,\overline{)444} & \\
\underline{420} & 70\\
24 & \\
24 & 4\\ \hline
0 & 74
\end{array}$$

In which one has 444 been renamed as (300 + 120 + 24)?

In which one has 444 been renamed as (360 + 60 + 24)?

In which one has 444 been renamed as (420 + 24)?

# Exercise Set 17

Divide.

1. $3\,\overline{)249}$
2. $4\,\overline{)284}$
3. $8\,\overline{)736}$
4. $5\,\overline{)365}$
5. $6\,\overline{)390}$
6. $7\,\overline{)518}$
7. $7\,\overline{)392}$
8. $6\,\overline{)378}$
9. $4\,\overline{)184}$
10. $3\,\overline{)252}$
11. $9\,\overline{)342}$
12. $8\,\overline{)664}$
13. $5\,\overline{)450}$
14. $3\,\overline{)291}$
15. $7\,\overline{)343}$
16. $9\,\overline{)711}$
17. $6\,\overline{)594}$
18. $7\,\overline{)679}$
19. $8\,\overline{)704}$
20. $9\,\overline{)801}$

## USING DIVISION IN PROBLEM SOLVING

There are 108 fruit trees in an orchard. There are 9 rows of trees with the same number of trees in each row. How can you find the number of trees in each row?

The information in the problem is:

There are 108 trees.
There are 9 rows.
Each row has the same number of trees.

The question we want to answer is:

How many trees are there in each row?

Let us form a mathematical sentence to show how the bits of information in the problem are related. Let $n$ represent the number of trees in each row.

$$9 \times n = 108, \quad \underline{\text{or}} \quad n = 108 \div 9.$$

In the mathematical sentence, 108 is the product, 9 is the known factor, and $n$ is the unknown factor. We can find $n$ by dividing 108 by 9. Your answer should be 12 so that

$$9 \times 12 = 108.$$

We now write an answer sentence:

There are 12 trees in each row.

In this problem about the trees, there are 108 trees in the set. The set of 108 trees is divided into 9 sets with the same number in each group. You found the number of trees in each of the 9 sets. This number was 12. You used division to find the number of trees in each set.

Now let us think of another problem. Suppose there are 822 dogs in a large dog show. An official tells us that there are cocker spaniels, poodles, collies, Irish setters, boxers, and German shepherds. Also, he tells us that in each breed there is the same number of dogs. How many dogs of each breed are in the show?

The information in the problem is:

> There are 822 dogs in the show.
>
> There are 6 breeds of dogs in the show.
>
> Each breed has the same number of dogs.

The question we want to answer is:

> How many dogs are there of each breed?

Let us form a mathematical sentence. Let $n$ represent the number of each breed of dog. $6 \times n = 822$, or $n = 822 \div 6$.

In the mathematical sentence, 822 is the product, 6 is the known factor, and $n$ is the unknown factor. We can find $n$ by dividing 822 by 6. We show the division in either one of these ways.

$$\begin{array}{r@{\,}r}
 & \underline{137} \\
 & 7 \\
 & 30 \\
 & 100 \\
6 & \overline{)822} \\
 & \underline{600} \\
 & 222 \\
 & \underline{180} \\
 & 42 \\
 & \underline{42} \\
 & 0
\end{array}
\qquad\qquad
\begin{array}{r@{\,}r|r}
6 & \overline{)822} & \\
 & \underline{600} & 100 \\
 & 222 & \\
 & \underline{180} & 30 \\
 & 42 & \\
 & 42 & \underline{7} \\
\hline
 & 0 & 137
\end{array}$$

The answer sentence is: There are 137 dogs of each breed in the show.

In this problem about dogs, there are 822 dogs in the set. There are 6 sets with the same number in each group. We found the number of dogs in each set by dividing 822 by 6.

## Exercise Set 18

Write mathematical sentences to help you solve the following problems. Solve them. Write an answer sentence for each problem.

1. The Jackson family is planning a 510 mile trip. If they have 5 days to travel, about how many miles must they travel each day?

2. Seven jets left the airport one day. Each had 128 passengers aboard. How many people left the airport by jet that day?

3. In Cub Scouts, John made a collection of 144 small shells. He put the same number in each of 6 boxes. How many did he put in each box?

4. Fran collected 126 leaves for a project in school. She mounted them on 9 large posters. How many leaves did she mount on each poster?

5. There are 189 Boy Scouts in 9 troops. If each troop has the same number of members, how many boys are in each troop?

6. Peggy's mother baked 186 cookies for a picnic. She packed the same number in each of three boxes. How many did she pack in each box?

7. Dick and Tom offered to make tickets for the puppet show. They made 139 tickets on Tuesday, 125 on Wednesday, and 127 on Thursday. How many tickets did the boys make together?

8. The restaurant had 2 dining rooms. One held 220 people, the other had room for 175 people. How many more people could eat in one dining room than in the other?

9. If 27 visitors are taken through a state capitol building in one group, how many visitors are taken through in 13 groups?

10. If one case of canned soup weighs 24 pounds, how much will 48 cases weigh?

11. A committee of 7 pupils collected 455 rocks while working on a class project. If each pupil collected the same number of rocks, how many rocks did each pupil find?

12. There are 9 boys in our Cub Scout den. The boys collected 477 toys during their yearly toy drive. If each boy collected the same number of toys, how many toys did each boy collect?

BECOMING MORE SKILLFUL IN DIVIDING NUMBERS

The fourth grade class had 1720 inches of string. They wanted to cut it into pieces, each 8 inches long. How many pieces will they have?

Mathematical sentence: $1720 \div 8 = n$ or $n \times 8 = 1720$

We can work this problem in several ways. Here are three ways.

Form I:

| (a) | (b) | (c) |
|---|---|---|
| 215 | 215 | |
| 5 | 5 | 215 |
| 60 | 10 | 5 |
| 100 | 100 | 10 |
| 50 | 100 | 200 |
| 8 )1720 | 8 )1720 | 8 )1720 |
| 400 | 800 | 1600 |
| 1320 | 920 | 120 |
| 800 | 800 | 80 |
| 520 | 120 | 40 |
| 480 | 80 | 40 |
| 40 | 40 | 0 |
| 40 | 40 | |
| 0 | 0 | |

Is (c) the shortest one of the 3 ways?

There are 215 pieces of string.

Form II:

(a)

```
8 )1720 |
    400 |  50
  ------|
   1320 |
    800 | 100
  ------|
    520 |
    480 |  60
  ------|
     40 |
     40 |   5
  -------------
      0 | 215
```

(b)

```
8 )1720 |
    800 | 100
  ------|
    920 |
    800 | 100
  ------|
    120 |
     80 |  10
  ------|
     40 |
     40 |   5
  -------------
      0 | 215
```

(c)

```
8 )1720 |
   1600 | 200
  ------|
    120 |
     80 |  10
  ------|
     40 |
     40 |   5
  -------------
      0 | 215
```

Is (c) the shortest of the three ways?

## Exercise Set 19

Find the missing factor.

1. 340 ÷ 4 = n

2. 567 ÷ 9 = n

3. 1435 ÷ 5 = n

4. 1056 ÷ 8 = n

5. 372 ÷ n = 6

6. 504 ÷ 7 = t

7. 474 ÷ m = 6

8. 420 ÷ 4 = p

9. 369 ÷ 3 = n

10. 2240 ÷ 4 = m

11. 5250 ÷ m = 7

12. 8280 ÷ 9 = t

13. 3616 ÷ 8 = n

14. 3560 ÷ 2 = n

15. 4362 ÷ 3 = k

16. 8960 ÷ 8 = s

17. 5761 ÷ 7 = m

18. 3768 ÷ 4 = t

19. 9384 ÷ 6 = p

20. 9639 ÷ 9 = s

## Exercise Set 20

Write mathematical sentences to help you solve the following problems. Solve them. Write an answer sentence for each problem.

1. If a plane travels 1675 miles in 5 hours, about how far does it travel in one hour?

2. How many doughnuts are there in 17 dozen?

3. For the school carnival the mothers put 600 pieces of homemade candy into bags. They put 5 pieces of candy in each bag. How many bags did they pack?

4. 720 ice cream bars were bought by the Dad's Club to treat the children of Baker School. There were 669 children present that day. How many extra bars were there?

5. A motorcycle traveled 234 miles on 6 gallons of gas. How far did it travel on one gallon?

6. A market put 1744 onions into bunches of 8 onions each. How many bunches were there?

7. A grocer ordered 726 bottles of soft drinks. They were delivered in cartons that held six bottles each. How many cartons were delivered?

## FINDING QUOTIENTS AND REMAINDERS

We can use the division process to solve problems like this one:

There are 306 people at the exhibit. There are to be 4 tours. How many people should go in each tour to have about the same number of people in each group?

Mathematical sentence: $306 = (4 \times n) + r$.

You may use either form.

Form I.

$$\begin{array}{r} \underline{76} \\ 6 \\ 70 \\ 4\,\overline{)306} \\ \underline{280} \\ 26 \\ \underline{24} \\ 2 \end{array}$$

Form II.

$$\begin{array}{r|l} 4\,\overline{)306} & \\ \underline{280} & 70 \\ 26 & \\ 24 & 6 \\ \hline 2 & 76 \end{array}$$

$$306 = (76 \times 4) + 2,$$

Each group should have 76 people. There are 2 people to join one or two of the groups.

In a mathematical sentence like

$$306 = (76 \times 4) + 2$$

we say that:

306 is the dividend.
76 is the quotient.
4 is the divisor.
2 is the remainder.

## Exercise Set 21

Find the numbers q and r must represent to make each sentence true.

1. $632 = (q \times 9) + r$

   $632 = (\_\_\_ \times 9) + \_\_\_$

2. $456 = (q \times 3) + r$

   $456 = (\_\_\_ \times 3) + \_\_\_$

3. $1576 = (q \times 5) + r$

   $1576 = (\_\_\_ \times 5) + \_\_\_$

4. $1242 = (q \times 8) + r$

   $1242 = (\_\_\_ \times 8) + \_\_\_$

5. $943 = (q \times 7) + r$

6. $1210 = (q \times 6) + r$

7. $421 = (q \times 3) + r$

8. $(q \times 4) + r = 3320$

9. $299 = (q \times 7) + r$

10. $151 = (q \times 4) + r$

11. $525 = (q \times 8) + r$

12. $373 = (q \times 5) + r$

## Exercise Set 22

Write mathematical sentences to help you solve the following problems. Solve them. Write an answer sentence for each problem.

1. 29 boys want to organize teams of 5 boys for a relay race. How many teams can be organized? How many boys will not race?

2. If the school custodian put 80 chairs into rows of 10 chairs each, how many rows could he make? Would there be any chairs not used?

3. Longfellow School bought 25 new bounce balls. Each of 6 classrooms are to share the balls equally. Any that are left will be kept for next year. How many balls will each room get?

4. "Polka for Three" is a dance done in groups of 3. How many groups can be made in a class of 32 children? How many children will not dance?

5. Mary is asking 30 girls to her party. How many tables must she have if she serves 4 at a table? How many girls will have to sit on the sofa to eat?

6. 271 reservations were made for a luncheon. How many tables would have to be set if 4 people were to be seated at each table?

## Exercise Set 23

1. A baker is to pack 1850 cupcakes for a school picnic. He will put 8 in each box. How many boxes will he order?

2. Each of 15 Girl Scouts sold 24 boxes of cookies. How many boxes were sold?

3. Six sheets of colored paper are needed for a booklet. How many booklets can be made from 500 sheets of colored paper?

4. The Parent Teacher Association of a school had 324 members last year and 296 members this year. How many more memberships are needed to reach last year's record?

5. In a school library there were 23 sets of readers. There were 35 books in each set. How many books were there in the 23 sets?

6. Bill planted 12 rows of tomatoes. There were 15 plants in each row. How many plants did Bill set out?

7. The children of Miller School are raising money to buy a television set which costs $350. They have collected $179. How much more money do they need?

8. Nancy is making some decorations for a party. She needs 360 white beads, 720 red beads, 180 green beads, and 45 yellow beads. How many beads does she need altogether?

9. If a jet travels 408 miles an hour, how far will it travel in 5 hours?

10. There were 385 tickets to be put in bundles of 8. How many bundles will there be? Will any tickets be left?

11. On a reading test, Mary read 284 words in 3 minutes. About how many words did she read in one minute?

12. A farmer packed 360 boxes of apples for shipping. Each box weighed 45 pounds. What was the weight of all the boxes?

13. 573 scouts who attended the Jamboree slept in tents which had 4 beds. How many tents would 573 scouts need?

14. 630 dancers attended the Folk Dance Festival. Into how many groups of 8 could they be divided for square dancing?

## Exercise Set 24

Find the numbers q and r must represent to make each sentence true.

1. 994 = (q × 8) + r
2. 889 = (q × 7) + r
3. 290 = (q × 9) + r
4. 493 = (q × 5) + r
5. 389 = (q × 4) + r
6. 534 = (q × 5) + r
7. 954 = (q × 4) + r
8. 588 = (q × 6) + r
9. 6769 = (q × 9) + r
10. 3626 = (q × 4) + r
11. 290 = (q × 9) + r
12. 5308 = (q × 7) + r
13. 7449 = (q × 8) + r
14. 3636 = (q × 8) + r
15. 2390 = (q × 6) + r
16. 1235 = (q × 5) + r
17. 2770 = (q × 3) + r
18. 477 = (q × 9) + r
19. 6792 = (q × 7) + r
20. 493 = (q × 3) + r

## Practice Exercises

I. Place the parentheses correctly to make these true mathematical sentences.

Example: 24 + 6 ÷ 3 = [illegible] ; (24 + 6) ÷ 3 = [illegible]

a) 6 × 9 + 4 = 58

b) 27 + 13 ÷ 4 = 10

c) 9 × 6 + 4 = 90

d) 7 × 8 + 8 = 112

e) 7 + 63 ÷ 9 = 14

f) 5 × 40 + 8 = 208

g) 7 × 9 − 4 = 35

h) 35 − 7 ÷ 4 = 7

i) [illegible]

j) [illegible]

II. [illegible]

[illegible]

III. Add:

25 496 589 32

38 447 9 200

49 588 889 4[illegible]

32 789 8889 3[illegible]

397 379 3709

Subtract:

7010 8300 610 9001

9258 7919 352 3729

Multiply:

358 868 69 98

7 8 38 89

Divide

9)12250 7)[illegible] 9)7749 6)3456

IV. In the chart below write a mathematical sentence that [illegible]

[illegible]

| Number Pair | Operation | Sentence | Result |
|---|---|---|---|
| [illegible] | addition | [illegible] | [illegible] |
| 1) [illegible] | [illegible] | | |
| 2) [illegible] | [illegible] | | |
| 3) [illegible] | [illegible] | | |
| 4) [illegible] | [illegible] | | |
| 5) [illegible] | [illegible] | | |
| 6) [illegible] | [illegible] | | |
| 7) [illegible] | [illegible] | | |
| 8) [illegible] | [illegible] | | |
| 9) [illegible] | [illegible] | | |
| 10) [illegible] | [illegible] | | |

V. By regrouping, find the unknown addend.

Example: 462 = 400 + 60 + 2 = 400 + 50 + 12
-157 = 100 + 50 + 7 = 100 + 50 + 7
300 + 0 + 5 = 305

a) 398
-219

c) 398
-213

b) 633
-563

d) 761
257

VI. Solve the following:

a) 35 × 27 = n f)

f) 126 ÷ 3 = n

b) n ÷ 5 = 405

g) 600 − n = 562

c) 9 × 847 = n

h) 876 + 889 = n

d) 352 + n = 900

i) 726 ÷ 8 = n

e) 27 + 5 + 8 = n

j) 9000 − 3482 = n

VII. Solve:

[illegible]

VIII. [illegible]

[illegible]

IX. Solve:

a) 63 × 80 = n

b) 40 + 23 + 17 = n

c) n + 4 = 40

d) 97 + n = 2005

e) 37 + 30 + 31 = n

f) 278 × 7 = n

g) 19 × 69 = n

h) 357 + 249 + 610 + 8 = n

i) 338 ÷ 5 = n

j) 201 ÷ 4 = n

Brain-twisters

1. Here is an imaginary operation called "cut". The symbol for cut is [illegible]. Try to find the meaning of [illegible] from these examples.

[illegible]

2. [illegible]

## Review

## SET I

### Part A

1. Use $=$ , $>$ , or $<$ to make these true statements.

Example: if $n + 2 = 7$ , then $n \underline{<} 7$

a) If $27 \div n = 9$ , then $n$ ___ $3$

b) If $n + 12 = 17$, then $n$ ___ $17$

c) If $n \times 15 = 45$ , then $n$ ___ $1$

d) If $50 - n = 50$ , then $n$ ___ $0$

e) If $128 \div n = 32$ , then $n$ ___ $4$

f) If $n \times 33 = 132$ , then $n$ ___ $33$

g) If $n \div 7 = 4$ , then $n$ ___ $4$

h) If $1407 + n = 2989$ , then $n$ ___ $1407$

i) If $143 = (2 \times 71) + n$, then $n$ ___ $2$

j) If $n - 6357 = 653$ , then $n$ ___ $6357$

2. Write a mathematical sentence to "undo" the following.

Example: $7 + 2 = n$ , $(7 + 2) - 2 = n$

a) $31 + 4 = n$

b) $12 \times 6 = n$

c) $15 \div 3 = n$

d) $423 + 172 = n$

e) $72 - 13 = n$

f) $64 \div 8 = n$

g) $125 - 25 = n$

h) $3 \times 3 = n$

i) $427 \div 7 = n$

j) $3592 - 1782 = n$

3. For each multiplication fact write two division facts.

Example: $2 \times 6 = 12$, $12 \div 6 = 2$ , $12 \div 2 = 6$

a) $6 \times 7 = 42$

b) $7 \times 8 = 56$

c) $8 \times 9 = 72$

d) $3 \times 8 = 24$

e) $6 \times 9 = 54$

f) $8 \times 6 = 48$

g) $7 \times 4 = 28$

h) $9 \times 5 = 45$

i) $4 \times 8 = 32$

j) $9 \times 7 = 63$

4. Write the correct words or numerals to complete this chart.

| Number Pair | Result | Operation | Result | Operation |
|---|---|---|---|---|
| a) 40, 25 | 65 | Addition | 15 | Subtraction |
| b) 72, 8 | 9 | ____________ | 576 | ____________ |
| c) 96, 8 | ___ | Subtraction | 12 | ____________ |
| d) 84, 23 | 1932 | ____________ | ___ | Addition |
| e) 369, 9 | 378 | ____________ | ___ | Division |
| f) 80, 12 | ___ | Addition | ___ | Subtraction |
| g) 45, 5 | 225 | ____________ | 9 | ____________ |
| h) 90, 9 | 81 | ____________ | ___ | multiplication |

5. B ⋃ E = {red, blue, white, green, purple}

B ⋂ E = { }

E = {green, purple}

What operation could you use to find the number of members in Set B? Name the members of Set B.

6. A ⋃ G = {2, 4, 5, 6, 3, 7}

A ⋂ G = {5, 6, 7}

G = {5, 6, 7}

Could you use subtraction to find the number of members in Set A? Name the members of Set A.

7. C = {2, 4, 6, 8}

O = {1, 3, 5, 7, 9}

Name the members of the Set C ⋃ O.

What operation could you use to find the number of members in C ⋃ O ?

8. Draw a polygon that is the union of

   a) 2 line segments

   b) 3 line segments

   c) 4 line segments

   d) 6 line segments

   e) 10 line segments.

9. How many vertices has each polygon in Problem 8 ?

10. Find the number that n represents in each of these. Example a is done for you.

    a) $53 + 22 + n = 89$, $53 + 22 = 75$, $89 - 75 = 14$, $n = 14$

    b) $24 + 30 + n = 79$

    c) $43 + n + 25 = 87$

    d) $n + 9 + 30 + 27 = 152$

    e) $798 + 9 + n = 1504$

    f) $59 + 497 + n + 7 = 1069$

    g) $34 + n + 11 = 68$

    h) $275 + 596 + n = 1716$

    i) $16 + n + 66 = 96$

    j) $n + 669 + 352 = 1021$

    k) $88 + 7 + n = 174$

11. Match each of the Ideas in Column I with a Model from Column II

Column I
Idea

1) Point
2) Line segment
3) Line
4) Ray
5) Plane
6) Simple Closed Curve
7) Polygon
8) Circle
9) Plane region

Column II
Model

a) My path when I walk all the way around the block and return to my starting point.
b) A stretched piece of string
c) The rim of a drinking glass
d) A football field
e) The tip of a compass
f) The edges of a piece of floor tile
g) The surface of a calm lake whose shores cannot be seen
h) The light from a distant star
i) A straight narrow road with no ends in sight

Part B

Write a mathematical sentence (or two sentences if necessary) for each problem and solve. Write an answer sentence.

1. The Polk Street bus makes three round trips every hour. How many minutes should one round trip take?

2. The school cafeteria charges 25 cents for lunch. How much money will a student need for lunches all week?

3. Eddie bought 6 tennis balls for $3.18. How much did one ball cost?

4. A dairy cow requires three acres of grazing land. How much land is needed for 175 dairy cows?

5. Mary's baby sister drinks 8 ounces of milk six times a day. How much milk will the baby drink in one week?

6. The class in Room 15 invited their parents to a puppet show. There were only forty-five chairs in the room and 72 parents came. How many parents had to stand?

7. One scout troup delivers 364 hand bills, another troop has 37 less to deliver. How many hand bills do both troops deliver?

Group Activities

Multiplication Quiz

Child (leader or quiz master) stands in front of the class and says, "I am thinking of two factors whose product is 42." Then he calls on class members.

Child in class group called on asks, "Are you thinking of 6 and 7 ?"

The leader replies yes or no as the case may be. A record is kept on the board of all combinations of numbers correctly given for review later.

If the leader passes a wrong combination he must sit down and a new leader is chosen.

## Review

## SET II

### Part A

1. Using the symbols $>$ , $<$ , or $=$ complete these to make true sentences.

a) $747$ _____ $319 \times 3$

b) $83 \times 7$ _____ $73 \times 8$

c) $576 \div 9$ _____ $32 \times 2$

d) $914 - 326$ _____ $22 \times 25$

e) $34 \times 19$ _____ $799 - 153$

f) $343 \div 7$ _____ $49 \times 6$

g) $3148$ _____ $232 \times 14$

h) $(7 \times 8) \times 2$ ___ $(12 \times 9) + 4$

i) $25 \times 25$ _____ $30 \times 20$

j) $(40 + 4) \times 4$ _____ $4 \times 44$

2. Tell whether each of the following is true or false.

a) $6 + 3 = 3 + 6$

b) $12 - 8 \neq 2 + 2$

c) $36 + 7 < 35 + 8$

d) $16 + 12 + 9 = 52 - 15$

e) $3 \times a$ is always $> 3 \times 2$

f) $(16 \div 2) \times 2 = 16 \div (2 \times 2)$

g) $7 \times 6 < 156 - 112$

h) $29 - 8 \neq 4 \times 7$

i) $4 \times 6 > 2 \times 11 \times 1$

j) $6 \times 5 \times 2 \neq 30 + 30$

3. Tell what operation is used and find $r$.

Example: $7 \times r = 42$ , division , $r = 6$

a) $23 = 14 + r$

b) $r = 5 \times 9$

c) $27 + 14 = r$

d) $16 + r = 34$

e) $r - 23 = 46$

f) $24 \times r = 120$

g) $56 \div r = 8$

h) $r = 42 - 16$

4. Write each division sentence as a multiplication sentence. Find the number $n$ represents.

Example: $64 \div 2 = n$, $2 \times n = 64$, $n = 32$

a) $832 \div 4 = n$

b) $273 \div 3 = n$

c) $568 \div 8 = n$

d) $4207 \div 7 = n$

e) $355 \div 5 = n$

f) $602 \div 7 = n$

g) $664 \div 8 = n$

h) $111 \div 3 = n$

5. Complete these to make them true sentences using words from this set of words: division, operation, multiplication, addends, subtraction, factor, addition, product.

a) We operate on two factors and get a __________.

b) The operation of __________ undoes subtraction.

c) We operate on two __________ and get a sum.

d) The operation of subtraction undoes __________.

e) To find an unknown addend we use __________.

f) We use division to find an unknown __________.

g) The operation of __________ produces a sum.

h) An __________ on numbers is a way of thinking about two numbers and getting one and only one number.

i) A product is the result of the operation of __________.

6. Complete these to make them true sentences. Find the product.

Example a is done for you.

a) $5 \times 14 = (5 \times 10) + (5 \times 4) = 70$

b) $6 \times 18 = (6 \times \underline{\quad}) + (6 \times 8)$

c) $9 \times 32 = (9 \times \underline{\quad}) + (\underline{\quad} \times 2)$

d) $7 \times 25 = (7 \times \underline{\quad}) + (7 \times 20)$

e) $5 \times 82 = (\underline{\quad} \times 80) + (5 \times \underline{\quad})$

f) $25 \times 6 = (\underline{\quad} \times 6) + (5 \times \underline{\quad})$

g) $100 \times 21 = (\underline{\qquad} \times 20) + (\underline{\qquad} \times 1)$

h) $32 \times 4 = (16 \times 4) + (\underline{\quad} \times 4)$

i) $1000 \times 13 = (1000 \times \underline{\quad}) + (\underline{\qquad} \times 3)$

7. Write each of the following using symbols.

Example: The number 8 increased by $y$, $8 + y$

a) The sum of $y$ and 6

b) The number $y$ added to 6

c) The number $y$ increased by six

d) Six more than the number $y$

Find the number represented by each of the above if $y = 7$.

8. Write each addition sentence as a subtraction sentence.

Find what number $n$ represents.

a) $40 + n = 68$

b) $36 + n = 39$

c) $n + 54 = 90$

d) $102 + n = 256$

e) $n + 69 = 534$

f) $452 + n = 931$

g) $384 + n = 731$

h) $465 + n = 534$

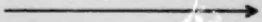
9. Match each word in Column I with a picture or a meaning in Column II

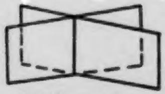
Column I

1) vertex
2) triangle
3) intersection
4) radius
5) quadrilateral
6) plane region
7) circle
8) triangular region
9) ray

Column II

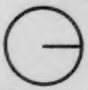
a) the union of a simple closed curve and its interior

b)


c)


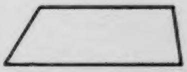
d) the study of space and location

e) the set of points that is the triangle and its interior

f)


g) the short way to name a line

h) the polygon that is the union of three line segments

i)


j) the common endpoint of two rays that are not on the same line

Part B

Write a mathematical sentence (or two sentences if necessary) for each problem and solve. Write an answer sentence.

1. Coach Lang paid 85 cents each for school softballs. How much does he pay for two dozen softballs?

2. How many ice cream cups can be bought for 90 cents if each cup costs 6 cents? _

3. There were 28 sixth grade girls, 32 fifth grade girls and 30 fourth grade girls at Lincoln School. How many girls were in the three grades?

4. For his model plane collection, Mark pays $1.29 for one model, $2.25 for another and $1.46 for another. What is the total cost of the models?

5. In the problem above, Mark had saved $3.29 and borrowed the remainder from his father. How much did he borrow?

6. Barbara can swim 120 yards in 5 minutes. How far can she swim in 20 minutes?

7. A sign in the bakery reads; cookies - 30 cents a dozen, donuts - 60 cents a dozen, chocolate cakes - 80 cents each. How much does it cost for two dozen cookies and a cake?

8. In the problem above, find the cost of two dozen cookies, two dozen donuts and a cake.

Individual Projects

1. Make up some operations and their symbols. Work at least 8 problems with each of your make-believe operations. Then put some examples on the board to see if your class can discover their meanings.

2. Arithmetic is only one kind of mathematics. There are at least 79 other kinds. Name five or more other kinds of mathematics. Make a chart for your classroom of the kinds you can find.

## Review

## SET III

Part : A

1. In the chart below, tell which property is illustrated by the number sentence at the left. Write the first letter of each word that names the property instead of writing the words. For example write A P A for Associative Property of Addition.

| Number Sentence | Property |
|---|---|
| a) 320 × 7 = (300 × 7) + (20 × 7) | |
| b) 641 × 29 = 29 × 641 | |
| c) 287 ÷ 7 = (280 ÷ 7) + (7 ÷ 7) | |
| d) 381 + (546 + 9) = (381 + 546) + 9 | |
| e) 250 ÷ 5 = (200 ÷ 5) + (50 ÷ 5) | |
| f) 37 + 504 = 37 + 504 | |
| g) 46 × 6 = (40 × 6) + (6 × 6) | |
| h) (23 × 7) × 18 = 23 × (7 × 18) | |

2. Fill in the symbol = or ≠ which makes each of the following a true sentence.

Example: 324 + 415 ≠ 748

a) 46 + 18 ___ 64

b) 303 + 235 ___ 538

c) 456 - 121 ___ 337

d) 87 + 344 ___ 431

e) 538 - 382 ___ 156

f) 534 - 273 ___ 271

g) 56 + 19 + 53 ___ 148

h) 941 - 327 ___ 624

i) 897 + 638 ___ 1535

j) 1962 - 1549 ___ 313

3. Place parentheses in these sentences to make them true.

Example: $4 \times 2 - 1 = 4$, $4 \times (2 - 1) = 4$

a) $23 + 2 \times 5 = 125$

b) $14 \div 2 \times 3 = 21$

c) $30 - 7 + 3 \neq 20$

d) $6 \times 2 - 5 = 7$

e) $5 + 3 \times 5 \neq 20$

f) $6 + 2 \times 3 \neq 12$

g) $16 \div 2 \times 4 \neq 2$

h) $135 \div 5 + 3 = 30$

i) $232 \times 6 - 5 = 232$

j) 123 [illegible]

4. Write each of these sentences using numerals and the symbols for "less than" and "greater than".

a) Three is less than five

b) Fifty-eight is greater than thirty

c) Eighteen is less than nineteen

d) Four hundred five is greater than five

e) Three tens are greater than twenty

f) One thousand twelve is less than two thousand

g) Seventy is greater than sixty-two

h) Nine hundred ten is less than ten hundred

i) Three hundred thousand is greater than three thousand

j) Forty-six is greater than twenty-six.

5. In the following exercises, use what you know about multiplying by 10 and 100 to get the answers.

Example: 4 x 364 = 1,456 , so 40 x 364 = 14,560

a) 80 x 117 = 9,360 , so 800 x 117 = ________

b) 5 x 766 = 3,830 , so 50 x 766 = ________

c) 9 x 36 = 324 , so 900 x 36 = ________

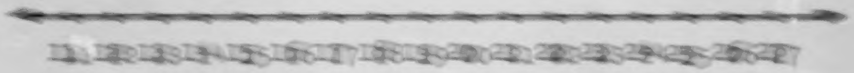
d) 30 x 592 = 17,760 , so 300 x 592 = ________

e) 8 x 125 = 1,000 , so 800 x 125 = ________

f) 3 x 987 = 2,961 , so 30 x 987 = ________

g) 12 x 91 = 1,092 , so 120 x 91 = ________

6.



Using the number line above find how many whole numbers are between these numbers.

a) 13 and 17

b) 12 and 13

c) 19 and 11

d) 19 and 25

e) 27 and 23

f) 21 and 20

g) 15 and 17

h) 12 and 26

7. Copy and complete these sentences.

a) A ray has ________ endpoint(s).

b) A triangle is the union of ________ line segment(s).

c) A line has ________ endpoint(s).

d) Space is the set of ________ point(s).

e) A line segment has ________ endpoint(s).

f) A radius is a line segment with ________ endpoint(s) on the circle.

g) A quadrilateral is the union of ________ line segment(s)

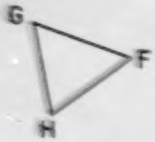
8. Match each word or symbol in Column I with a picture in Column II.

Column I

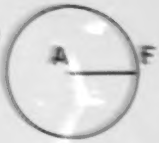
1) $\overrightarrow{AB}$
2) $\angle CDE$
3) triangle
4 $\overline{GH}$
5) $\angle BAC$
6) triangular region
7) $\overline{AF}$
8) circle
9) quadrilateral

Column II

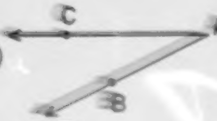
a)



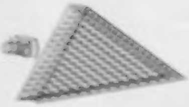
b)



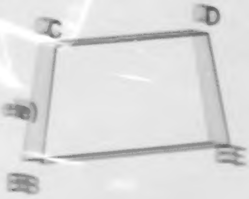
c)



d)



e)



Part B

Write a mathematical sentence (or two sentences if necessary) for each problem and solve. Write an answer sentence.

1. The Clark family traveled 387 miles in 9 hours. How many miles did they average each hour?

2. During Public Schools Week, 1,162 people visited Pine Grove School, 1,219 visited Sleepy Hollow School, and 1,094 visited Inland Valley School. How many people visited the three schools?

3. Dean and Gail have stamp collections. Dean has 364 stamps. He needs 37 more to have as many stamps as Gail. How many stamps does Gail have?

4. The thirty-four children in Room 7 were making bird pictures. The bulletin board would hold 4 dozen pictures. How many children would need to make two pictures?

5. A jet liner averages 449 miles an hour between Los Angeles and St. Louis. The trip takes four hours. How many air miles is it between the two cities?

6. How much more do I pay for two shirts that cost $2.15 each than for one shirt that costs $3.29?

7. The price of potatoes is 5 pounds for 29 cents. What is the cost of twenty pounds of potatoes?

Group Activity

Tic, Tac, Toe

The objects of the game are speed and accuracy in addition. This is a racing game. Each child draws intersecting line segments as shown. The sum is announced by the teacher. The children put single digit addends in squares so that each row gives the sum.

Example: Sum is 13.

| | | |
|---|---|---|
| 6 | 5 | 2 |
| 4 | 3 | 6 |
| 3 | 6 | 4 |
| 7 | 4 | 2 |

Individual Project

Use only polygons to make an interesting drawing. See how many polygons your classmates can identify.

# Chapter 8

# RECOGNITION OF COMMON GEOMETRIC FIGURES

## REVIEW OF TRIANGLE AND QUADRILATERAL

### Thinking Together

1) a) What name is given to a polygon which is the union of three line segments?

The three line segments are called the sides of the triangle.

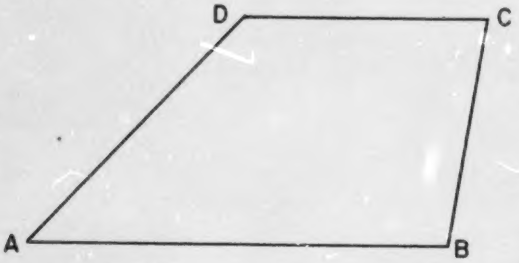
b) What is the common endpoint of any two sides of a triangle called?

c) What are the endpoints of the line segments of a triangle called?

d) How many sides and vertices does a triangle have?

2) What is a polygon which is the union of four line segments called?

We call the four line segments of the quadrilateral the sides of the quadrilateral. Below is a drawing of a quadrilateral with sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$.



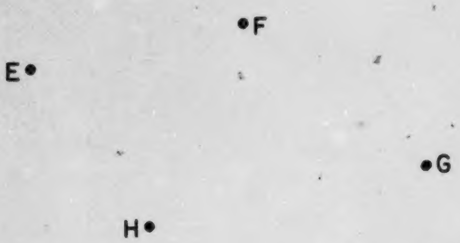
Let us recall that quad suggests four. In the figure shown above four angles are formed. They are $\angle DAB$, $\angle ABC$, $\angle BCD$, and $\angle CDA$. There are some points of these angles that are not points of the quadrilateral because angles are made up of rays.

The vertex of one of the angles is called a vertex of the quadrilateral.

The vertices of these four angles are called the vertices of the quadrilateral.

## Exercise Set 1

1. a) Four points are marked below. Trace these points on a sheet of paper and label them as shown.



b) Draw $\overline{EF}$, $\overline{FG}$, $\overline{GH}$, $\overline{HE}$.

2. On the sheet of paper on which you drew the figure for exercise 1 write answers to the following questions:

a) Do these segments form a quadrilateral?

b) Name the sides of this quadrilateral.

c) Name a vertex of the quadrilateral.

d) Name the vertices of this quadrilateral.

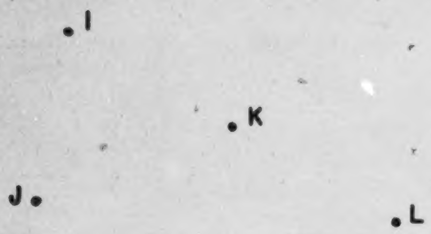
e) Color the interior of the quadrilateral.

3. Go back to exercise 1. Trace the 4 points again on a sheet of paper and label them as shown.

a) Draw $\overline{EG}$, $\overline{FH}$, $\overline{EH}$, and $\overline{FG}$.

b) Do these segments form a quadrilateral? Why?

4. a) Four points are marked below. Trace these points on a sheet of paper and label them as shown.



b) Draw $\overline{IJ}$, $\overline{JL}$, $\overline{LK}$, and $\overline{KI}$.

5. On the sheet of paper on which you drew the figure for exercise 4, write answers to the following questions.

a) Is your figure a union of four line segments?

b) Do $\overline{IK}$ and $\overline{KL}$ lie on the same line?

c) Is your figure a quadrilateral? Why or why not?

d) Is your figure a polygon?

e) What is a name for your figure?

6. Mark three points (not all on the same line) on your paper. Label them P, Q, and R. Draw $\overline{PQ}$, $\overline{QR}$, and $\overline{RP}$.

a) Is your figure a polygon?

b) Is your figure the union of three line segments?

c) What is a name for your figure?

d) What is a triangle?

## COMPARING LINE SEGMENTS

### Thinking Together

1) Henry says he thinks the line segment represented by the edge of his desk is longer than the line segment represented by the bottom edge of the door. Bill thinks differently. They have only a long piece of string. How can they find out which segment is longer?

2) Bill says, "I can take this long piece of string and hold it at one corner of your desk. Then we can extend the string along the edge to the other corner. Let us hold this string so that it represents the edge of the desk."

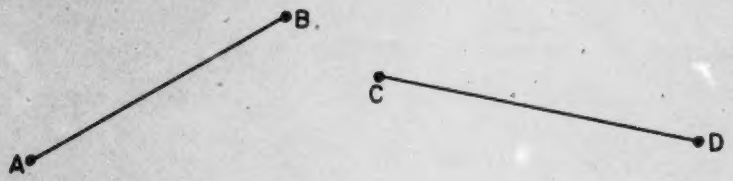
"Henry, you hold your end of the string at one corner of the bottom edge of the door. I place the string along the edge leading to the other corner. Suppose the string does not reach the other corner. Then the segment represented by the bottom edge of the door is longer than that represented by the edge of your desk. If the string goes beyond the other corner, then the edge of the door is shorter than the edge of the desk. If the string matches exactly the bottom edge of the door from corner to corner, the line segments represented are congruent.

Bill says the line segment represented by the edge which runs from the top to the bottom of the door is longer than the line segment connecting the corner of his desk to the teacher's desk. How can Bill decide which is longer?

You can always compare line segments which are represented by objects if you have a piece of string that is long enough.

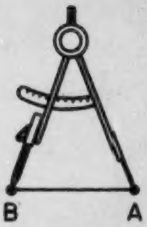
Another way to compare line segments is to use a compass.

If you had only a compass and you wished to decide which of the represented line segments below is the longer, how would you do it?



3) Follow the directions and you will learn how a compass may be used to compare the line segments represented above. On a separate sheet of paper trace the above figure.

a) $\overline{CD}$ is part of $\overrightarrow{CD}$. Extend $\overrightarrow{CD}$ to the edge of the sheet of paper.



b) Set the metal tip of the compass on A and the pencil tip of the compass on B.

c) Without changing the setting of your compass, place the metal tip at C as if you were going to draw the circle with center at C.

d) Draw just enough of the circle to intersect $\overrightarrow{CD}$.

e) Label the intersection T. If T is between C and D, then $\overline{AB}$ is shorter than $\overline{CD}$. If D is between C and T, then $\overline{AB}$ is longer than $\overline{CD}$. If the point of intersection is D, then $\overline{AB}$ is congruent to $\overline{CD}$ (or $\overline{AB}$ and $\overline{CD}$ are congruent segments).

4) a) Compare the segments with a piece of string.

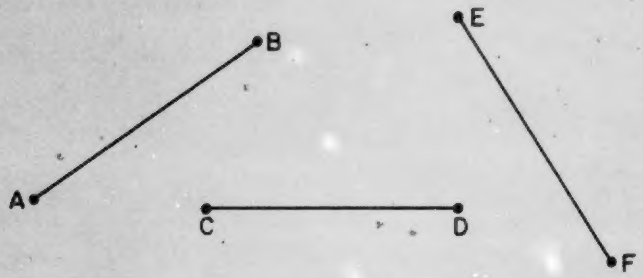
b) Which way of comparing do you think is better here?

5) Trace the three line segments. Compare their lengths. Use a compass.

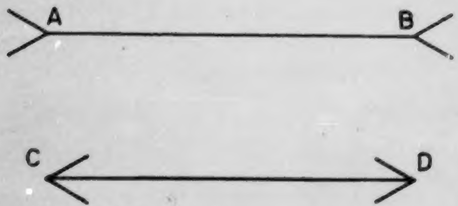
a) Name the shortest line segment.

b) Name the next shortest.

c) Name the longest.

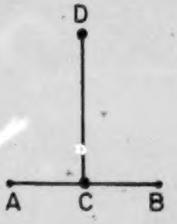


6) Look at $\overline{AB}$ and $\overline{CD}$ below. Which appears to be longer? Use your compass to compare $\overline{AB}$ and $\overline{CD}$. Which is longer?



7) Compare $\overline{AB}$ and $\overline{CD}$.

Which is longer?



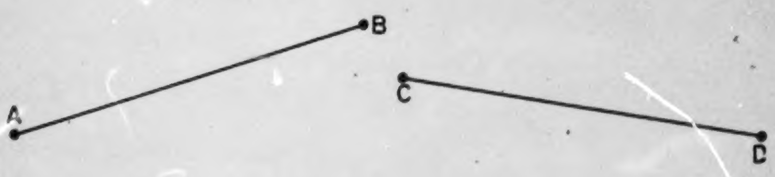
Exercises 6 and 7 shows us that sometimes we must use an instrument to compare segments. We cannot trust our eyes alone.

8) The line segments represented below are congruent.

a) Show that they are congruent by using your compass.

b) Place a sheet of thin paper over $\overline{AB}$ and trace it.

c) Move the tracing so that the dot marked A covers point C. Can you make the dot marked B cover D? The tracing of $\overline{AB}$ matches the drawing of CD exactly, endpoint for endpoint.



Of course, we have not actually moved $\overline{AB}$. We have moved a drawing of it.

9) If someone asked you to compare segments, which method would you use?

## ISOSCELES AND EQUILATERAL TRIANGLES

### Thinking Together

A triangle which has at least two sides congruent to each other is called an isosceles triangle.

A triangle which has three sides congruent to each other is called an equilateral triangle.

1. Which of the triangles below is an isosceles triangle? Which is equilateral?

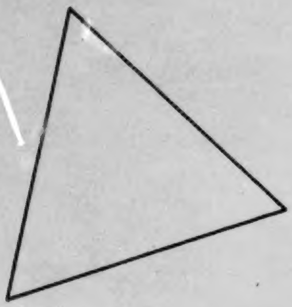


Figure 1

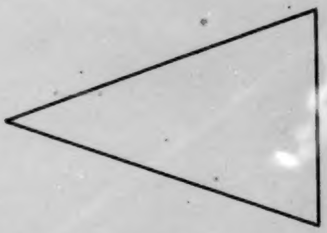
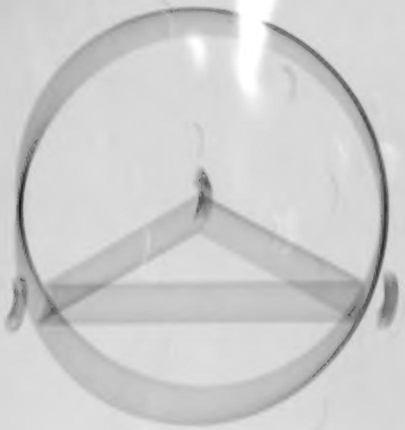


Figure 2

2. Are there any isosceles triangles suggested by the edges of the models in your classroom?
3. Are there any equilateral triangles suggested by the edges of the same models?
4. Name some things on which you see isosceles triangles represented.
5. Name some things on which you see equilateral triangles represented.

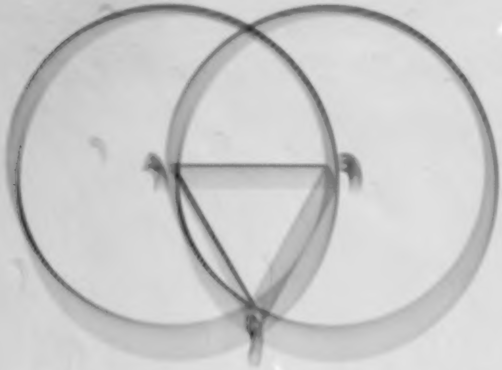
## Exercise Set 2

2. a) Draw an isosceles triangle using only a compass, pencil, and a straightedge. This picture may help you follow the directions of your teacher.



b) Is AC congruent to AB? Why?

2. a) Draw an equilateral triangle using only a compass, pencil, and a straightedge. This picture may help you follow the directions of your teacher.



b) Is $\overline{AB}$ congruent to $\overline{BC}$ and to $\overline{AC}$? Why?

3. Draw an isosceles triangle which is not also an equilateral triangle. Make one of its sides congruent to $\overline{DE}$.

D ———————————— E

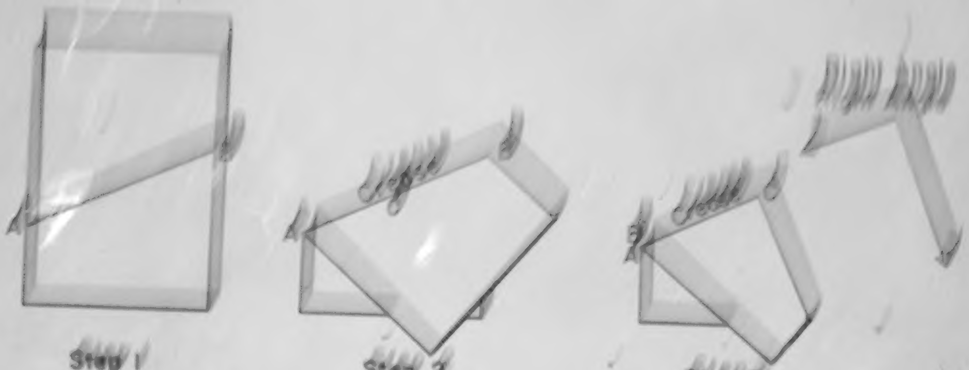
4. Draw an equilateral triangle. Make each of its sides congruent to $\overline{DE}$ of exercise 3.

# RIGHT ANGLES

## Thinking Together

We have learned that an angle is the union of two rays which are not on the same line. The two rays must have a common endpoint. By folding paper we are going to represent an angle which is called a _right angle_.

Fold a sheet of paper. It is not necessary to have the edges even. The crease represents a line segment. Now fold the paper again so that the edges of the first crease line up exactly. The intersection of the two creases is the vertex of an angle. The creases represent part of the rays of an angle. [illegible] The angle that is represented is called a right angle.



Step 1 Step 2 Step 3

Does this page itself suggest a model of a right angle?

Name some other models of right angles in your classroom.

We shall use the right angle model to draw a right angle having a chosen ray as one of its rays. Let the ray represented below be the chosen ray. Draw a ray like this on a sheet of paper.

1. a) Label the endpoint of the ray as A.
   b) Place the folded paper model of the right angle so that the vertex is at A and one of the creases lies along the ray.
   c) Trace along the other crease from the vertex.
2. There are two possible right angles which can be represented on the paper if the instructions above are followed. Draw both of them.

We shall use our model to compare angles with a right angle. Suppose we wish to compare ∠BAC, represented in Figure 1, with a right angle. On another sheet of paper copy ∠BAC. Then trace the right angle ∠BAD with $\overrightarrow{AB}$ as one of its rays. Draw it so that D is on the same side of $\overleftrightarrow{AB}$ as C. Notice that $\overrightarrow{AC}$ falls between $\overrightarrow{AB}$ and $\overrightarrow{AD}$. We shall say, therefore, that ∠BAC is smaller than ∠BAD. So ∠BAC is smaller than a right angle.

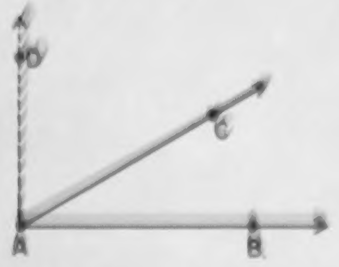


Figure 1

Suppose instead of the picture of the previous figure, we have the picture below. Here $\overrightarrow{AD}$ is between $\overrightarrow{AC}$ and $\overrightarrow{AB}$. So we say that

$\angle BAC$ is greater than a right angle.

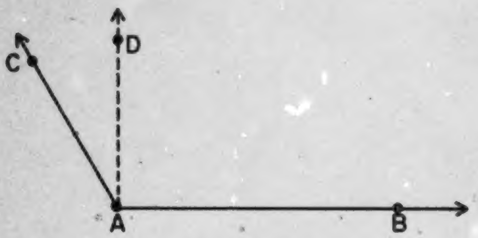


Figure Two

Of course, if C is on $\overrightarrow{AD}$, then $\angle BAC$ is a right angle.

## Thinking Together

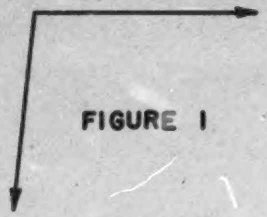


FIGURE 1

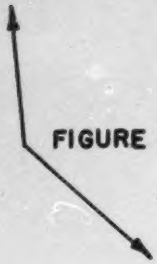


FIGURE 2

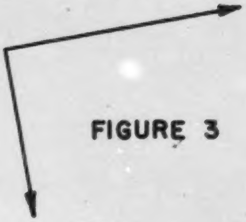


FIGURE 3

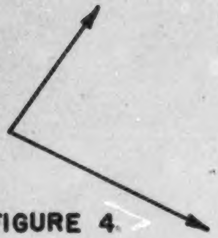


FIGURE 4

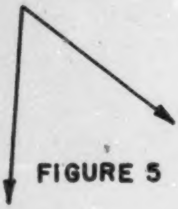


FIGURE 5

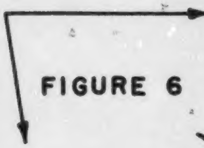


FIGURE 6

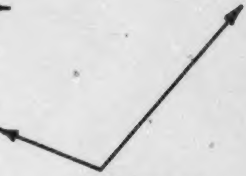


FIGURE 7

1. In this picture seven angles are represented. Look at each angle carefully. Without using a right angle model, name those angles which you think are right angles.

2. Without using a right angle model, name those angles which seem to be greater than a right angle.

3. Without using a right angle model, name those angles which seem to be smaller than a right angle.

4. Now check the figures with a folded paper model of a right angle to see if your answers are correct.

5. Are your answers in Exercise 4 the same as your answers in Exercises 1 - 3?

6. You see how an angle may be compared with a right angle if the angle has been represented by a drawing. Suppose an object suggests an angle. How would you compare this angle with a right angle? Find some object in your classroom which suggests an angle and compare it with your model of a right angle.

7. Are the angles represented by the edges of your desk right angles?

8. a) Do the hands of a clock ever suggest right angles?
   b) Name a time when they suggest an angle less than a right angle.
   c) Name a time when they suggest an angle greater than a right angle.

9. Sue wants to know if the angle the hands of the clock suggest when the time is 12:10 is larger or smaller than a right angle. She has a tracing of a right angle on thin paper. How might she use this tracing to decide whether or not the angle is larger or smaller than a right angle?

10. Name some other objects which suggest right angles.

## RECTANGLES AND SQUARES

### Thinking Together

If each of the four angles of a quadrilateral is a right angle, we say that the quadrilateral is a rectangle.

1. Can you find edges of your book which represent a rectangle? Check with your right angle model.

2. Do the edges of this sheet of paper represent a rectangle? Check with your right angle model.

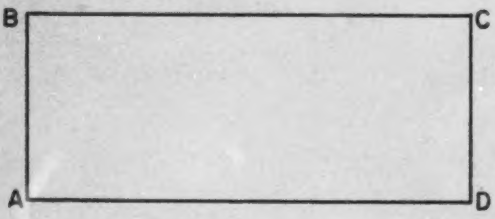
3. Can you find any models in your classroom which represent a rectangle? Check with your right angle model.

4. Name some objects at your home which suggest rectangles.

5. How could you draw a rectangle using your right angle model?

## Exercise Set 4

1. a) Is the quadrilateral represented below with vertices A, B, C, and D a rectangle? Use your model of a right angle.



   b) Is $\overline{AB}$ congruent to $\overline{BC}$?

   c) Is $\overline{AB}$ congruent to $\overline{DC}$?

   d) Is $\overline{BC}$ congruent to $\overline{AD}$?

   e) Is $\overline{BC}$ congruent to $\overline{CD}$?

2. Which of these statements is true?

   a) A rectangle has two pairs of congruent sides.

   b) All four sides of every rectangle are congruent.

   c) A rectangle has four right angles.

3. Make a copy of $\overline{AD}$ of exercise 1. Using $\overline{AD}$ as one side, see if you can draw another rectangle which looks different from rectangle ABCD of exercise 1.

## Thinking Together

A rectangle with all its sides congruent to one another is called a square.

1. Below is a representation of a square. Check the angles with your right angle model and the sides with your compass.

   a) Are all the sides congruent to each other?
   b) Are all the angles right angles?

2. Name some objects in your classroom which suggest squares.

3. Name some objects in your home which suggest squares.

4. Is every square a rectangle?

5. a) Is every rectangle a square?
   b) Are some rectangles also squares?

BRAINTWISTER

Draw a square using only the folded paper model of a right angle, a compass, and a pencil.

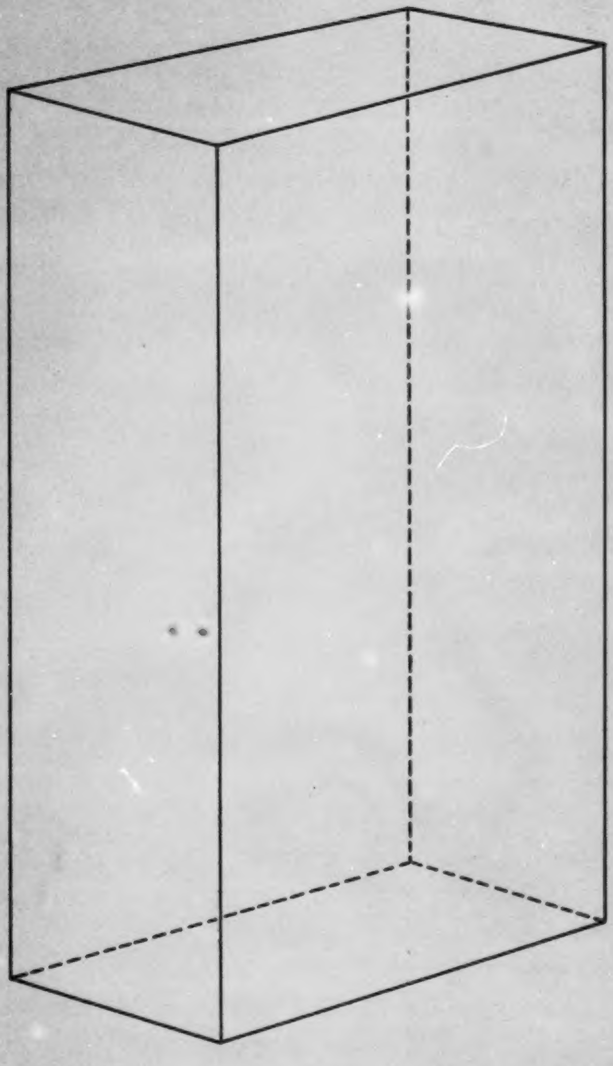
## Thinking Together

We are going to look at some objects. The surfaces of these objects represent sets of points in space. These sets have names which you will find below the pictures of the objects.

The objects are called models because they represent sets of points. Parts of the surface of some of these objects remind us of parts of planes because they are flat.

A closer look at these flat parts shows that one suggests a triangular region (a triangle and its interior). Another flat part of a model suggests the union of a quadrilateral and its interior. Still others remind us of circular regions (circles and their interiors).

Not all parts of the surfaces are flat. For example, the sphere, the cylinder, and the cone have parts of surfaces which are not flat.



Rectangular Prism

## Rectangular Prism

### Thinking Together

Let us look closely at this model of a rectangular prism. Any one of the flat parts of the surface suggests a quadrilateral and its interior. The set of points on the flat part is called a face. The union of all the faces is called a rectangular prism. A rectangular prism consists of the entire surface of the model but not the inside of the model.

1. Look at a face.

   a) Trace with your finger the edge of a face. The edges represent the sides of what figure?

   b) Show with your finger the vertices of the rectangle.

   c) Mark with your pencil three points in the interior of the rectangle.

2. How many faces does the rectangular prism have? The edges of the model represent line segments. These line segments are the intersection of two different faces and are called edges of the rectangular prism.

3. a) Trace an edge with your finger.

   b) Show two faces whose intersection is this edge.

   c) Count the number of edges of the rectangular prism. How many are there?

The corners of the model represent points. Each such point is called a vertex of the prism; it is also a vertex of each of the three quadrilaterals which come together at the corner. The plural of the word "vertex" is "vertices", so we can speak of one vertex and several vertices.

4. a) Mark a vertex of the rectangular prism on your model with a pencil.

   b) What three quadrilaterals have this point as a vertex?

   c) How many vertices does the rectangular prism have?

5. Name some other objects which represent a rectangular prism.

6. If a wooden block were hollow, would it still represent a rectangular prism?

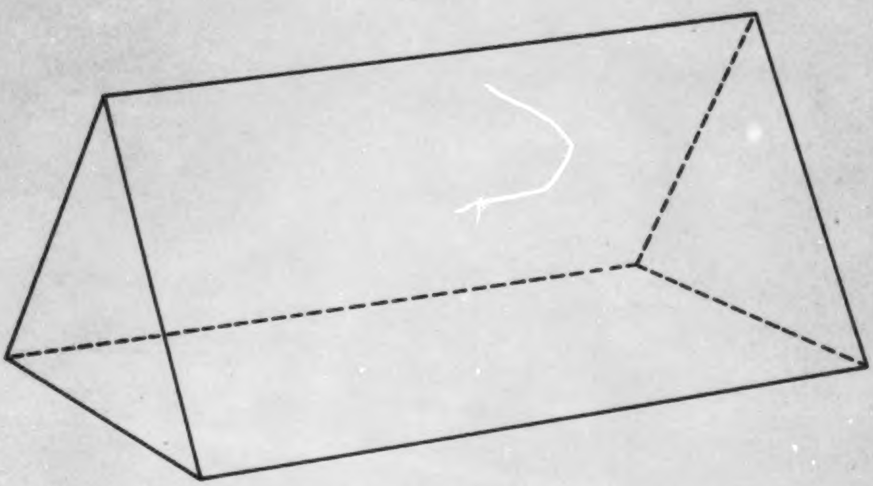
## Exercise Set 2

1. Complete the following sentences, using a separate sheet of paper.

    a) A rectangular prism has ______ faces. Each face represents a rectangular region.

    b) The rectangular prism has __________ edges and ______________ vertices.

2. Is a rectangular prism hollow?



Triangular Prism

Triangular Prism

## Thinking Together

The model we will study next represents a triangular prism. As in the rectangular prism, the flat parts of the model will represent plane regions which are called faces. The triangular prism is the union of the faces. (Notice that a triangular prism, as we have defined it, is "hollow".)

1. Are all the faces of the triangular prism the union of rectangular regions?
2. Indicate a face which does not represent a rectangular region.
3. What does this face represent? We call a face which represents a triangle and its interior a triangular face or a triangular region.
4. How many triangular faces are there?
5. How many rectangular faces are there?

The triangular prism also has line segments which are the intersections of two faces and are called edges. The endpoints of the edges are called vertices of the prism. They are represented by the corners of the model.

6. How many edges does a triangular prism have?
7. How many vertices does a triangular prism have?
8. Name some objects which represent a triangular prism.

## Exercise Set 3

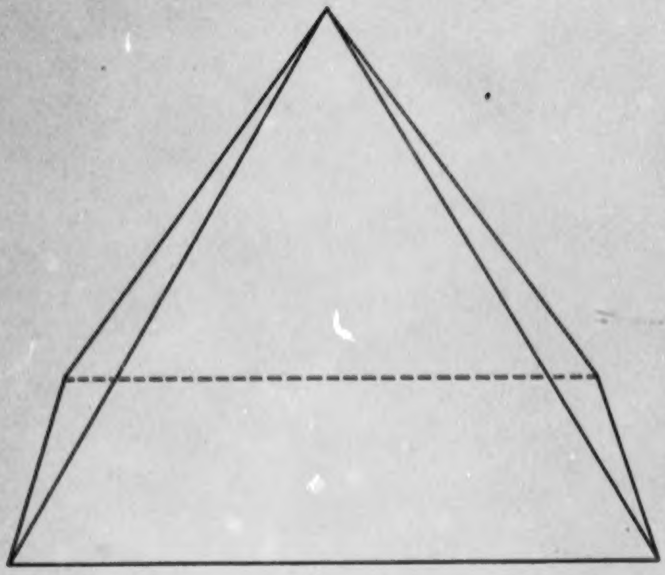
1. Look at your model and write on a separate piece of paper the words that complete the following sentences.

a) A triangular prism has _____ triangular faces.

b) A triangular prism has _____ faces.

c) A triangular prism has _____ edges.

d) A triangular prism has _____ vertices.



Pyramid

## Thinking Together

Look at the model of the pyramid pictured on page 446. It is made up of flat parts only, like those of a prism. These suggest parts of planes, and as in the case of the prism, they will be called faces. The pyramid is the union of these faces. (Notice that a pyramid, as we have defined it, is "hollow".) Faces next to each other intersect in a line segment which has endpoints called vertices.

1. a) How many triangular faces does this pyramid have?
   b) Are there any faces on this pyramid which are not triangular? Trace this face with your finger.
   c) How many faces are there on this pyramid?
   d) How many vertices are there on this pyramid?
   e) Put your finger on a vertex which is the intersection of three faces.
   f) Show a vertex which is the intersection of four faces.
   g) How many edges does this pyramid have?
   h) Trace with your finger four edges which intersect at a vertex.

2. Name some objects which suggest a pyramid.

Cylinder

Cylinder

## Thinking Together

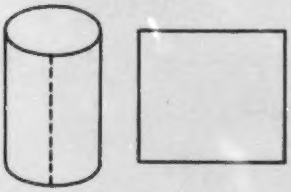
Let us look at the model of the cylinder which you have brought to class.

1. a) Are all parts of the surface of the model flat?

   b) What do the flat parts represent?

   c) Are the circular regions the same size?

Remove the top and bottom of a box which is the model of a cylinder. Make one straight cut as shown in the picture. Spread out the part just cut so that it lies flat on the desk.



2. a) What geometric figure is represented?

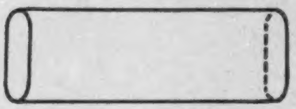
   b) Does this suggest how you might construct a model of a cylinder?

   c) Put the parts of the box together again so that it is a model of a cylinder.

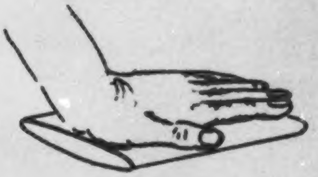
3. Name some objects which represent cylinders or parts of cylinders.

4. Not all cylinders are like the oatmeal boxes or the cans that you brought. Your models are examples of a special kind of cylinder called a circular cylinder. Circular cylinders have circular regions for bases.

Many cylinders do not have circular regions for bases. Look at this picture of a cylinder. Its bases are not circular regions.

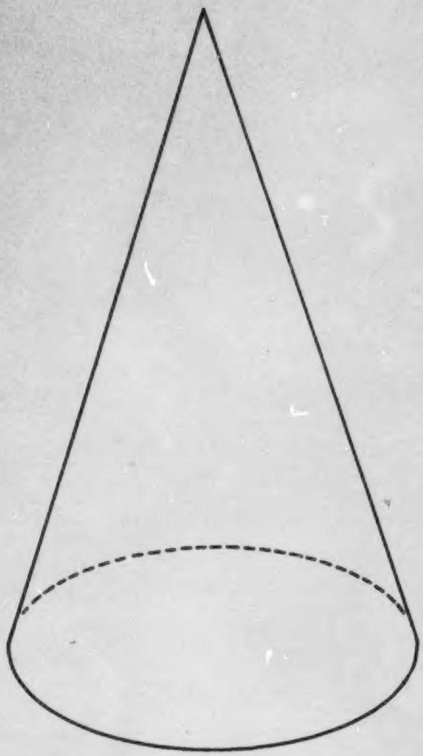


Take a can which is a model of a circular cylinder. Take out both bases. Press it down like this:



Think of bases that are oval regions being put on the can. Now we have a model of a cylinder which is not a circular cylinder.

Try to find a model of a cylinder which is not a circular cylinder. Sometimes toothbrushes come in this kind of container. Would part of the front or the back fork of a bicycle be an example of a cylinder?



Cone

## Thinking Together

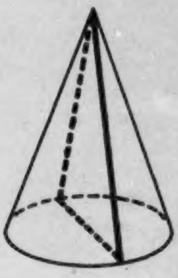
We can see from the photographs that the model of a cone is different from that of a cylinder.

1. Describe ways in which the model of the cone differs from that of the cylinder.

2. a) What geometric figure does the flat part suggest?
   b) Place the model of the cone so that the flat part is on a sheet of paper and trace around the edge. What is represented?

3. What is represented by the tip of a model of a cone? The point represented by the tip is called the vertex of the cone.

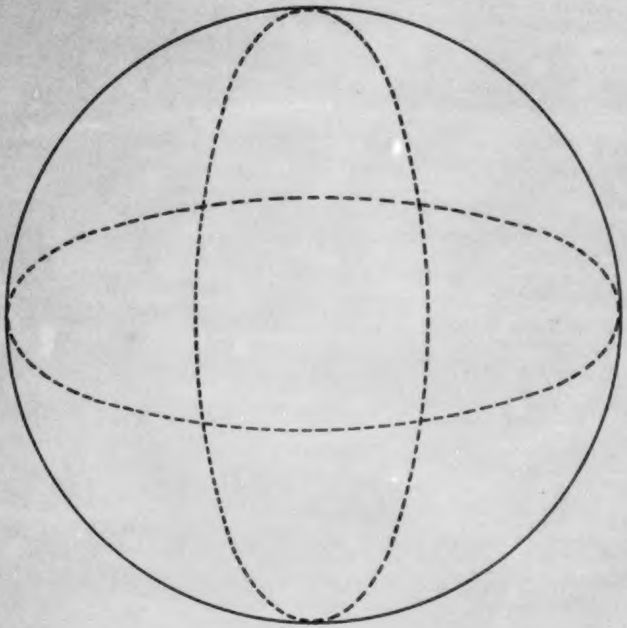
4. Suppose the model of a cone is cut in half so that the cut passes through the vertex and divides the circular region into parts of equal size.



   What will the cut edge represent? The cut edge is shown as the heavy line segments in this drawing.

5. Name other objects which suggest cones or parts of cones.

6. Is a cone hollow?



Sphere

## Thinking Together

The surface of a ball or a globe is a model of a sphere.

1. Look at the shadow of a ball on the sidewalk when the sun is directly overhead, or look at the shadow made by a ball with a light directly over it.

   What does the edge of the shadow look like?

2. Cut a model of a sphere into two pieces of equal size.

   What geometrical figure do the edges of each half of the sphere represent?

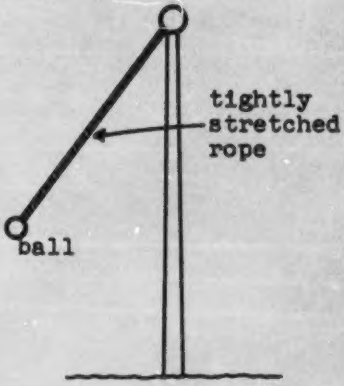
3. Place one of the halves of the sphere on a flat surface so that the cut edge is against the flat surface. Trace around this edge.

   What does the drawing represent?

4. Do you think that all the points of the half sphere are the same distance from the center of the circle that is represented by the edge?

5. Does this suggest that there is a point inside the sphere that we might call the center?

6. Think of a tetherball. When the rope is tightly stretched, the ball is always the same distance from the point where the rope is attached. As the ball moves around into different positions, do you think the point where the rope is attached to the ball goes through points on a sphere?



7. How would you describe a sphere?

8. Would the description of the sphere as "the set of all points in space that are the same distance from a chosen point called the center" be a good description?

9. How does this description differ from that of a circle?

10. Is a sphere hollow?

## EDGES AND FACES

### Thinking Together

1. Do any edges of a triangular prism form a rectangle? Check with your right angle model.

2. Do any edges of a pyramid form a rectangle? Check with your right angle model.

3. How many faces of the rectangular prism have edges which form a rectangle?

4. How many faces of the triangular prism have edges which form a rectangle?

5. How many faces of the pyramid have edges which form a rectangle?

6. Look at the models of a rectangular prism, a triangular prism, and a pyramid. Copy the following table on a sheet of paper and fill in the blank spaces.

| | Number of Faces | Number of Vertices | Number of Edges | Number of faces plus number of vertices minus number of edges |
|---|---|---|---|---|
| Rectangular prism | | | | |
| Triangular prism | | | | |
| Pyramid | | | | |

a) Add the number of faces and the number of vertices of the rectangular prism. Subtract the number of edges from this sum. What is your answer?

b) Do the same for the triangular prism and the pyramid. What is your answer in each case?

c) Look at other geometric figures made up of portions of planes. Is the number of faces plus the number of vertices minus the number of edges the same for these geometric figures ?

d) Do you think you will always get the same answer if you add the number of faces and the number of vertices, then subtract the number of edges of any geometric figure which is the union of parts of planes?

e) Write a mathematical sentence for your answer of 6d.

# Chapter 9

# LINEAR MEASUREMENT

## COMPARING SIZES

### Questions About Size

### Exploration

How many of the questions you ask and other people ask begin, "How much?" "How many?" "How far?"

We say: "How many pupils are there in your class?"

"How long is the hall outside your classroom?"

"How far is it to New York?"

To answer these questions, we use numbers.

There are 32 pupils in the class.

The hall is 320 feet long.

It is 750 miles to New York.

Answers to some of these questions can be found by counting. Other answers can not be found just by counting. Why?

Which of these can be found just by counting?

1. The size of your class
2. The height of the tallest boy in your class
3. The size of your family
4. The size of your classroom
5. The length of a book shelf
6. The size of a rock collection
7. The size of the smallest rock in the collection
8. The population of your town
9. How hard the wind is blowing
10. The size of a bicycle wheel

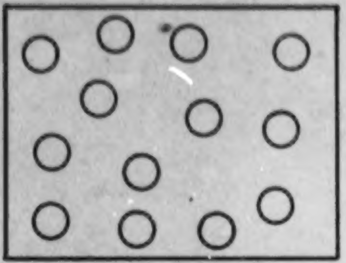
Think of a rock collection. What can you tell about the rock collection by counting? Is there something you cannot tell about the size of the rock collection by counting?

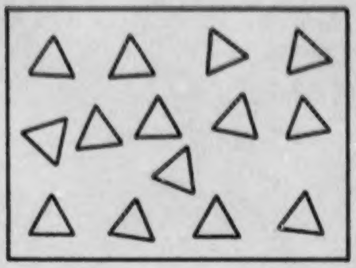
You can tell the size of a rock collection by counting, because each rock is a separate thing. A rock may be large or small, but it is one rock.

You cannot tell the size of any single rock by just counting. It is one rock, but it may be large or small.

## COMPARING SIZES WITHOUT COUNTING

### Exploration





Without counting, how can you
tell which set has more members?

1. A scout brings a bag of candy bars to a den meeting. What is an easy way to tell, without counting, whether there are more scouts or more candy bars?

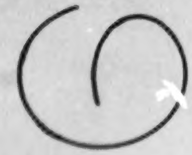
2. In a school storeroom there is a supply of desks and a supply of chairs. How can you tell which supply is larger?

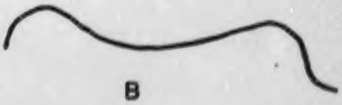
3. How can you tell how the attendance at a movie compares with the seating capacity?

4. How can you tell whether there are more hot dogs or more buns?

You can compare the sizes of single objects whose size can not be found by just counting.

5. Here are pictures of two pieces of rope.


A


B

Which rope do you think is longer,

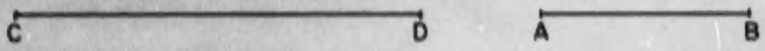
A or B?

If you had the ropes instead of the pictures,

how could you tell which is longer?

To compare curves like those in the pictures, lay a string along each curve. Then straighten out the strings. What geometric figures do the stretched-out strings represent?

6. Suppose you wish to compare $\overline{AB}$ and $\overline{CD}$.



To prove to someone that $\overline{CD}$ is longer than $\overline{AB}$, stretch a string on $\overline{AB}$. With your fingers, hold the points of the string that fall on A and B. Move the string and place it on $\overline{CD}$ with one endpoint on C. Is the other endpoint on D? Since it is between C and D, $\overline{CD}$ is longer than $\overline{AB}$. How is comparing segments, to see which is the longer, something like matching groups of separate objects to compare the sizes of the groups without counting?

7. There are many things which are enough like our idea of a line segment so that we can think of them as line segments; for example,

a stretched string,
some pencil marks on paper,
the edge of a table,
a pencil.

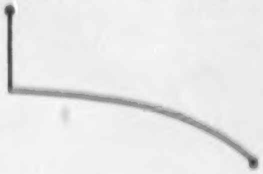
Name several other things which we might think of as line segments.

8. Remember that between any two points in space there is just *one* line segment. If we have any two objects which we *think of* as *points*, we can also think of them as endpoints of a line segment. This is what we mean when we say:

> the distance between the two tips of a compass,
>
> the distance between the earth and a star,
>
> the distance from home plate to second base,
>
> the height at which an airplane flies.

Name several other ways we think of line segments by thinking of objects as their endpoints.

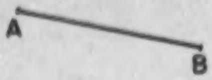
9. The way you go to school is probably not much like a line segment. A picture of it might look like this:

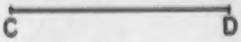


We can still talk about the distance you travel in going to school. Your path can be thought of as a *curve*, but not as a *line segment*. What does distance or length mean for curves? We think of the curve as represented by a piece of wire or string. Then we imagine straightening out the wire or string to represent a line segment.

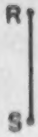
## Exercise Set 1

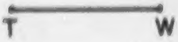
Use strings to compare the line segments and other curves. Copy each sentence below the figures and write "longer" or " shorter" in the space.



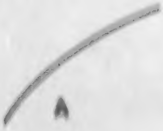


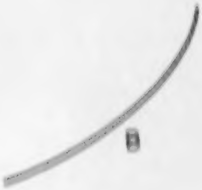
$\overline{AB}$ is __________ than $\overline{CD}$.



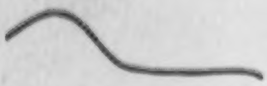


$\overline{RS}$ is __________ than $\overline{TW}$.

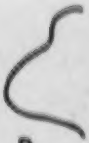




Curve A is __________ than curve B.

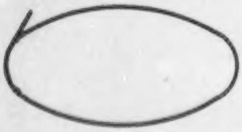


C

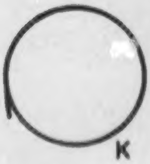


D

Curve C is ______ than curve D.

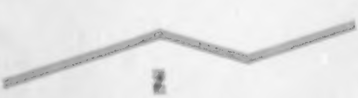


M

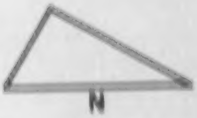


K

Curve M is ______ than curve K.



Z



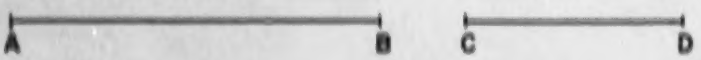
N

Curve Z is ______ than curve N.

## USING A COMPASS TO COMPARE SEGMENTS

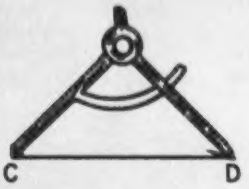
### Exploration

Is $\overline{AB}$ longer than $\overline{CD}$?

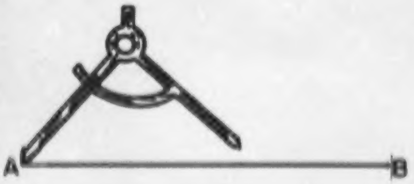


Recall how you have used a compass to compare segments.

Place the points of your compass on C and D.

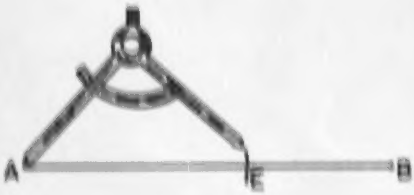


Without changing your compass, place the sharp point on A.



Draw a small part of a circle which intersects $\overline{AB}$.

Label the intersection E.



E is between A and B, so $\overline{AB}$ is longer than $\overline{AE}$.

$\overline{AE}$ has the same length as $\overline{CD}$, so $\overline{AB}$ is longer than $\overline{CD}$.

## Exercise Set 2

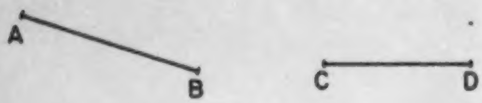
Trace the following figures on your paper.

Use your compass or string to help you compare segments.

Copy and answer the question for each exercise.

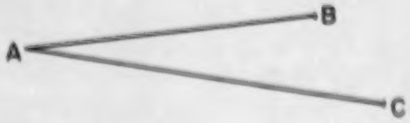
Color the unmatched part of the longer segment.

1.



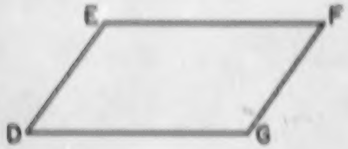
Which is longer, $\overline{AB}$ or $\overline{CD}$?

2.



Which is longer, $\overline{AB}$ or $\overline{AC}$?

3.



Which is longer, $\overline{DE}$ or $\overline{EF}$?

# MEASURING SEGMENTS

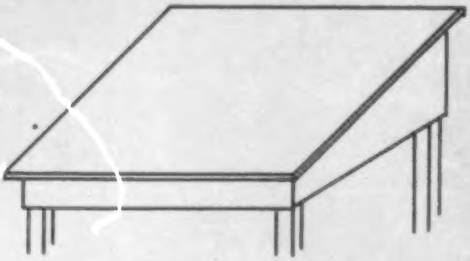
## Measuring a Segment

### Exploration

You know that you cannot tell the size of a segment by just looking at it and counting. You can tell how long it is by comparing it with some other segment.

How can you tell how long your desk is?

What geometric figure does the edge of your desk represent?



1. Take your pencil.

   Lay it along the edge of your desk, with one end on the corner.

   Put your finger on the desk at the other end of the pencil.

   Lay the pencil down again from that point.

   How many pencils long is the edge of your desk?

   Give your answer to the nearest whole number.

2. People often use their hands to tell how long a segment is. Spread your right hand so the fingers are as far apart as possible. Place your right hand with your thumb on the left corner of your desk. See how many hand-spans long your desk is.

The segment represented by your pencil or your hand-span is called your unit of measure.

The number of pencil-lengths or hand-spans it took to cover the edge of your desk is the measure of your desk. A measure of your desk may be 7.

To name a length, we use both the measure and the unit of measure. A length of your desk may be 5 hand-spans.

## Exercise Set 3

Copy and complete each of the sentences.

1. Our family drinks 3 quarts of milk each day.
   a. The unit of measure is ____________.
   b. The measure is ________________.
   c. The amount of milk is ____________.

2. My dog weighs 18 pounds.
   a. The unit of measure is ____________.
   b. The measure is ________________.
   c. Its weight is ________________.

3. My desk is 9 chalk-pieces long.
   a. Its length is ________________.
   b. Its measure is ________________.
   c. The unit of measure is ____________.

In Exercises 4-9 use the words length, measure, or unit of measure, so that each sentence will make sense.

4. It takes 6 chalk-pieces to cover the edge of my desk.
   Its ______________ is 6.
5. My desk is 4 pencils long.
   Its ______________ is 4 pencils.
6. My desk is 4 hand-spans long.
   The ______________ is the hand-span.
7. The ______________ in hand-spans is 5.
8. The ______________ I used is a segment represented by one pencil.
9. The edge of my desk has a __________ of 4 pencils.

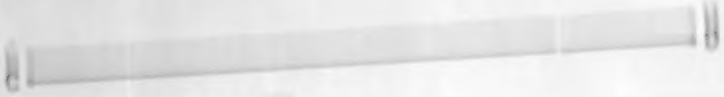
# USING A COMPASS TO MEASURE LINE SEGMENTS

## Exploration

We want to find the measure of $\overline{CD}$.

We use our compass to help us

measure a line segment.



Our unit of measure is $\overline{AB}$.

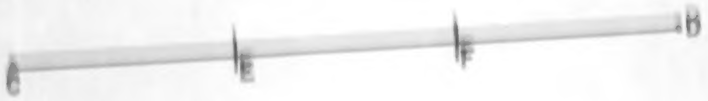
Here is $\overline{AB}$.

We lay off the unit $\overline{AB}$ on $\overline{CD}$.

We lay off $\overline{AB}$ on $\overline{CD}$ three times.

We label the intersections E and F.

See the picture below.



We say the measure of $\overline{CD}$ is 3.

We write: $m\ \overline{CD} = 3$

What is the measure of $\overline{CE}$?

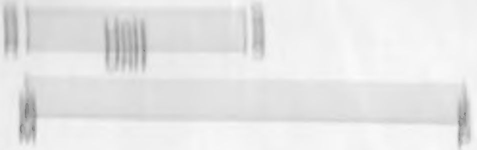
What is the measure of $\overline{CF}$?

What is the measure of $\overline{ED}$?

## EXERCISE SET [illegible]

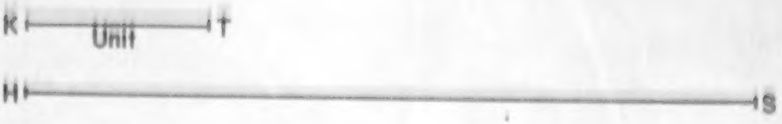
Copy each of the segments in exercise 1 and 2. Find the measure of each segment.

1.



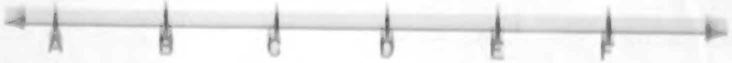
m $\overline{MP}$ = ____

2.



m HS = ____

3. Tell the measures of the segments in the figure.



m $\overline{AE}$ = ____

m $\overline{AD}$ = ____

m $\overline{BE}$ = ____

m $\overline{CF}$ = ____

m $\overline{BF}$ = ____

m $\overline{AF}$ = ____

4. Write the name of any segment which has the measure given. Use the unit and figure for Exercise 3.

m ____ = 2

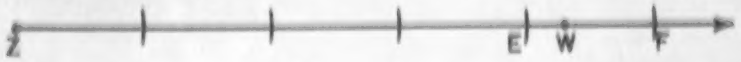
m ____ = 4

## Exploration

We want to find the measure of $\overline{ZW}$.

We use $\overline{AB}$ as the unit of measure.

This picture shows how we can find the measure of $\overline{ZW}$, using $\overline{AB}$ as unit.



How many times can you lay off the unit on $\overline{ZW}$?
m $\overline{ZE}$ = 4. Is m $\overline{ZW}$ larger than 4? Why?
m $\overline{ZF}$ = 5. Is m $\overline{ZW}$ smaller than 5? Why?
The measure of $\overline{ZW}$ is between 4 and 5 units.

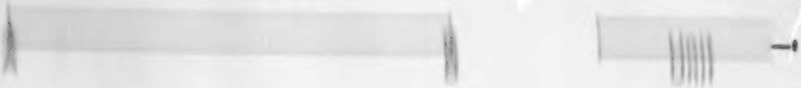
Since W is nearer to E than F, we say

m $\overline{ZW}$ = 4, to the nearest unit.

## Exercise Set 5

Trace the figures in each of the following exercises. Find the measures of the segments to the nearest unit.

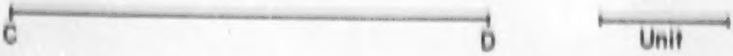
1.



The length of $\overline{AB}$ is greater than ___ units but less than ___ units.

m $\overline{AB}$ = ___ (to the nearest unit)

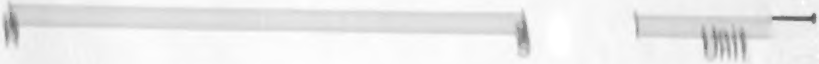
2.



The length of $\overline{CD}$ is greater than ___ units but less than ___ units.

m $\overline{CD}$ = ___ (to the nearest unit)

3.

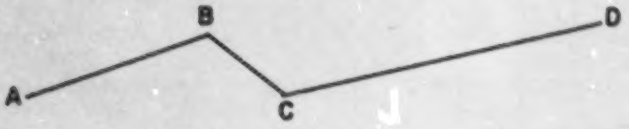


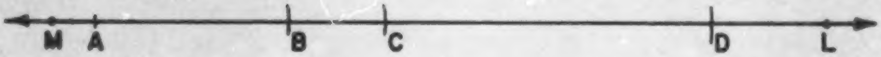
The length of $\overline{RS}$ is greater than ___ units but less than ___ units.

m $\overline{RS}$ = ___ (to the nearest unit)

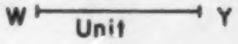
**BRAINTWISTER:**

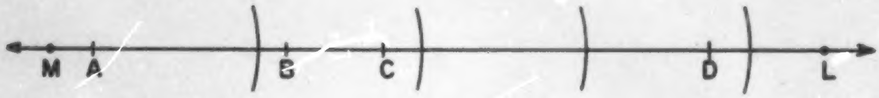
On $\overleftrightarrow{ML}$ draw a segment whose length is the length of the curve ABCD.





Now find the measure of curve ABCD. Use $\overline{WY}$ as unit.





The length of curve ABCD is greater than ____ units but less than _____ units.

m ABCD = _____ (to the nearest unit)

# USING STANDARD UNITS OF LENGTH

## Exploration

Suppose a team of boys from your school were going to play a game of baseball with a team from another school. If the other team brought a baseball so large and heavy you could hardly lift it, what would you say? You would probably say, "We will not play with that baseball. It is not the standard size and weight." What does that mean? Can you find out the standard size of a baseball?

You have been using units of measure which were not of "standard" size. Now we shall use standard units, which are used by a great many people and which always mean the same amount. The size of a standard unit is set by law. Your encyclopedia contains information about the National Bureau of Standards in Washington, D. C.

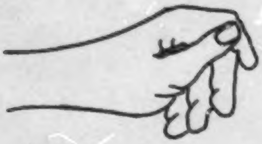
In which of the following sentences are standard units used?

1. He is as strong as an ox.
2. Put in a pinch of salt.
3. We get $\frac{1}{2}$ pint of milk for lunch.
4. The corn is knee high.
5. I used to live a day's journey from here.

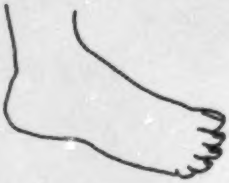
You are familiar with many standard units. Name some of these units.

Primitive people had little need for standard units. If the caveman liked the size of his neighbor's spear, he could borrow the spear and copy its length. Or, he could think, "When the spear is held with one end on the ground, the other end reaches my shoulder." Then, he could make a spear with that same length.

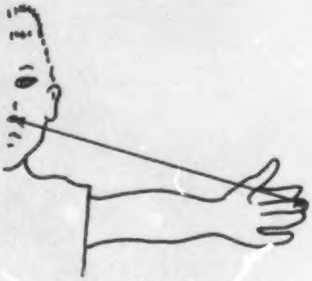
Many of the standard units which we use came from units which were not standard. These were used by people long ago. Many of them came from using parts of the body.



An inch came from the use of a part of a finger as a unit of length. Can you find a part of your finger which is about an inch long?



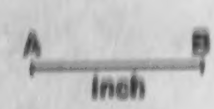
A foot came from the length of a person's foot. Is your foot shorter or longer than a foot ruler?



A yard was at one time thought of as the distance from the tip of a person's nose to the tip of his middle finger, when his arm was held straight out from the shoulder. Is the distance from your nose to the tip of your finger as long as a yardstick?

Here is a model of a standard unit.

You have often used this unit.

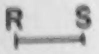


It is the inch.

Name some objects you measure in inches.

Here is a model of another standard unit.

This unit may be new to you.



It is the centimeter.

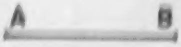
If you lived in France, or in many other countries, you would use this unit segment instead of the inch. Scientists in all countries use this unit.

# Exercise Set 6

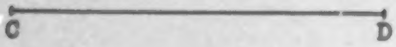
Trace each figure in exercises 1-8 on your paper. Use your compass or string to find the measures of the segment.

Copy and complete the statement for each exercise.

For exercises 1-4, use $\overline{AB}$ as the unit segment. Give your answer to the nearest inch.

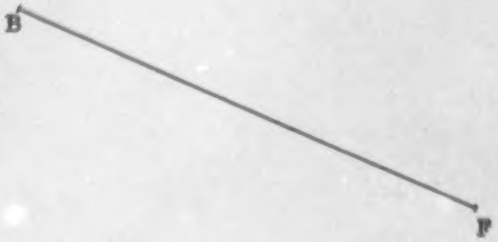


1.



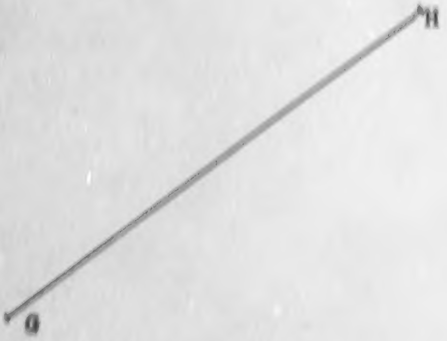
$m\,\overline{CD}$ = _____, in inches

2.



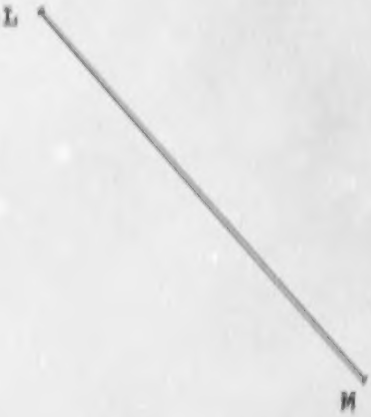
$m\,\overline{EF}$ = _____, in inches

3.



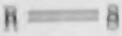
$m\,\overline{GH}$ = _____, in inches

4.



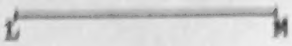
$m\,\overline{LM}$ = _____, in inches

For exercises 5-8, use $\overline{RS}$ as the unit segment.



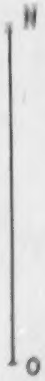
Give your answer to the nearest centimeter.

5.



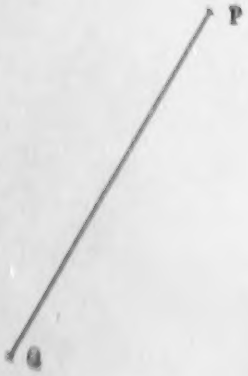
m $\overline{LM}$ = _____,
in centimeters.

6.



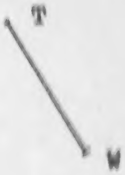
m $\overline{NO}$ = _____,
in centimeters.

7.



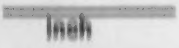
m $\overline{PQ}$ = _____,
in centimeters.

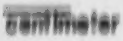
8.

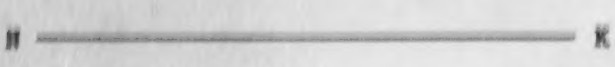


m $\overline{TW}$ = _____,
in centimeters.

9. Make two copies of $\overline{HK}$.
Find the measure of $\overline{HK}$
in inches.
Then find its measure in
centimeters.

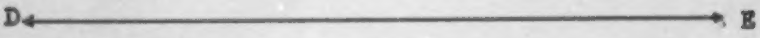






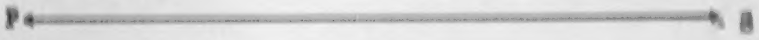
m $\overline{HK}$ (in inches) = ____
m $\overline{HK}$ (in centimeters) = ____

10. Draw $\overleftrightarrow{DE}$.



On $\overleftrightarrow{DE}$ draw a segment whose measure, in inches, is 3.
Label it $\overline{FH}$.

Draw $\overleftrightarrow{FS}$.



On $\overleftrightarrow{FS}$, draw a segment whose measure in centimeters
is 3 Label it $\overline{ZW}$.
m $\overline{FH}$ (in inches) = ____
m $\overline{ZW}$ (in centimeters) = ____
Is $\overline{FH}$ congruent to $\overline{ZW}$?

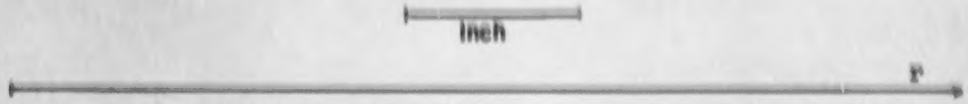
SCALES OF MEASURE

Making Inch and Centimeter Scales

## Exploration

When you are measuring segments, it is convenient to have a scale.

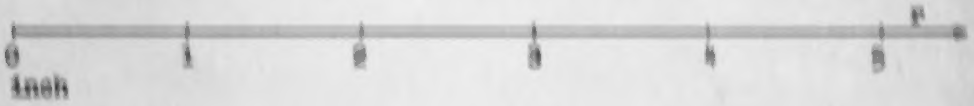
Follow these directions to make an inch scale on ray r. (Sometimes we name a ray by a single letter as $\vec{r}$.)



1. Write zero below the endpoint of $\vec{r}$. Then use your compass to copy the inch unit segment, beginning at 0. Label the other endpoint 1.

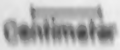
2. Copy the inch unit segment again, beginning at 1. Label the other endpoint 2.

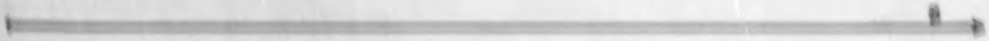
3. Continue copying the unit segment until you have copied it five times. Label the endpoints of the unit segments. Write "inch" below 0. Does your drawing now look like this one?



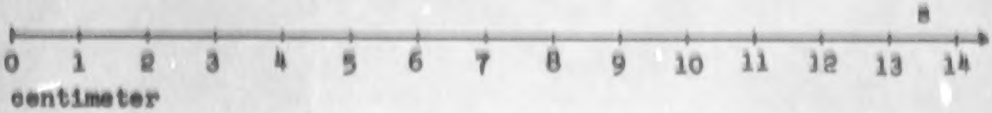
Save your scale.

4. Trace ray a. On ray a make a centimeter scale.



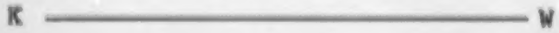


5. What is the largest number on your centimeter scale?
What is the largest number on your inch scale?
Does your centimeter scale look like this one?



Save your centimeter scale, too.

6. You can use your scales to find the measure of a segment in inches and in centimeters.
Copy and find the measure of $\overline{KW}$, in inches.



Place your compass with the points on K and W. Without changing your compass, place the sharp point on the zero-point of the inch scale and the pencil point on the ray. What point on the scale is nearest the pencil point? What is its number? What is the measure of $\overline{KW}$ in inches?

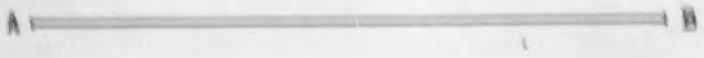
7. Find the measure of $\overline{KW}$ in centimeters. Use your compass and scale.

## Exercise Set 7

Using your inch scale and your centimeter scale, find the measures of these segments.

Find the measures in inches and in centimeters.

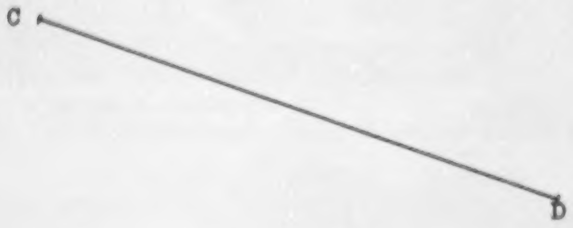
1.



m $\overline{AB}$ (in inches) = ____

m $\overline{AB}$ (in centimeters) = ____

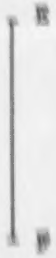
2.



m $\overline{CD}$ (in inches) = ____

m $\overline{CD}$ (in centimeters) = ____

3.

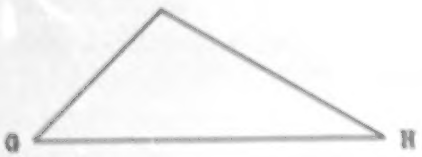


m $\overline{EF}$ (in inches) = ____

m $\overline{EF}$ (in centimeters) = ____

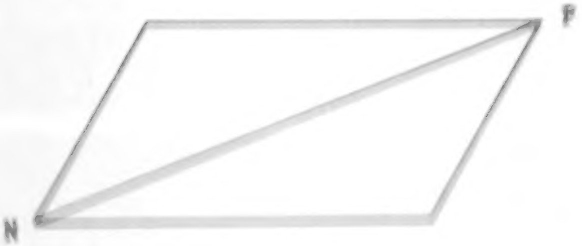
these figures, the endpoints of one segment are named.

nd the measure of the segment, in centimeters.



m $\overline{GH}$ (in centimeters) = ____

5.



m $\overline{NP}$ (in centimeters) = ____

## THE INCH SCALE AND THE CENTIMETER SCALE

### Exploration

You have made an inch scale and a centimeter scale.

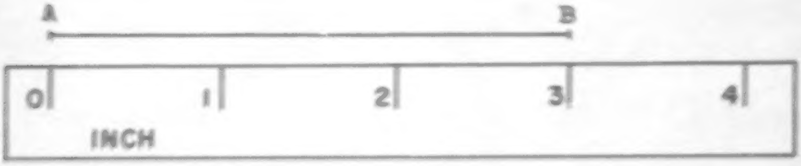
You have used these scales to find measures of line segments.

You have found measures of line segments in other ways, too.

Now we will use only the scales of measure.

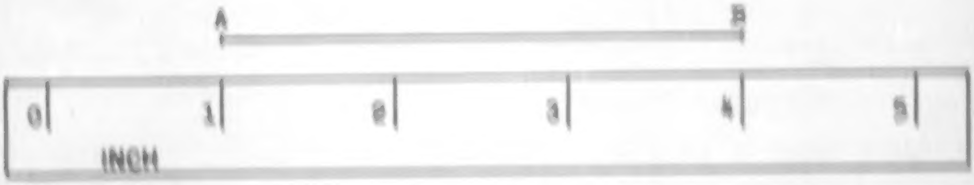
We find the length of $\overline{AB}$.

1. We can place our scale along $\overline{AB}$ like this:



2. Here is another way of placing our scale.

Why would you place the ruler in this way?



3. Are there other ways to place the ruler to find the length of $\overline{AB}$?

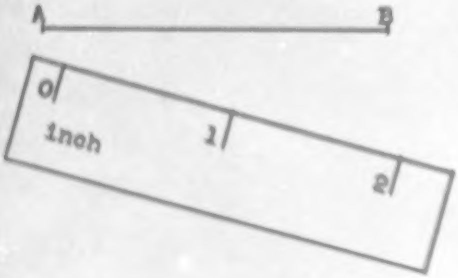
The length of $\overline{AB}$ is 3 inches.

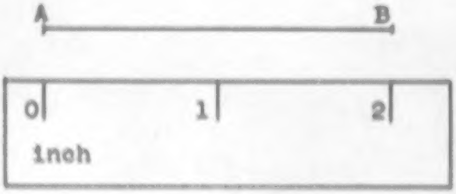
We write $m\,\overline{AB} = 3$.

We read: The measure of $\overline{AB}$ is 3.

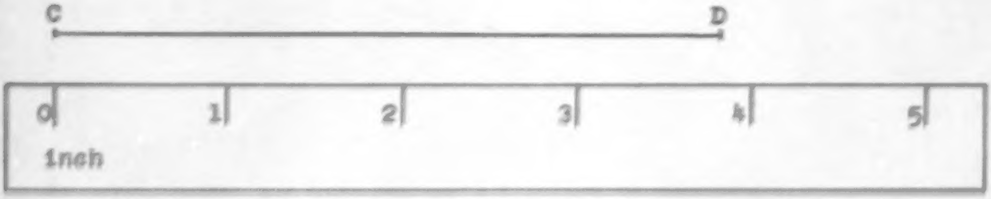
When you used the scale to find the measure of $\overline{AB}$ you used it as a ruler. A ruler is a straightedge with a scale on it.

4. Which of these drawings shows the correct way to place a ruler to measure a segment? Why?





5. Find the length of $\overline{CD}$, to the nearest inch.



Place the scale along $\overline{CD}$ as in the picture.
What point on the scale is below C?
Point D lies between what two points on the scale?
Is D closer to 3 or 4 on the scale?
What is the length of $\overline{CD}$, to the nearest inch?

Give two other ways to use the scale to measure $\overline{CD}$.

6. Here are pictures of some line segments.

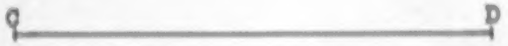
Find the length of each segment to the nearest inch.

Then, find the length of each segment to the nearest centimeter.

The abbreviation for "centimeter" is "cm."

Write your answers for each as has been done in the first one.

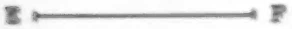
a)



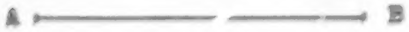
Length of $\overline{CF}$ is __3__ in., to the nearest in.

Length of $\overline{CD}$ is __7__ cm., to the nearest cm.

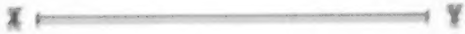
b)



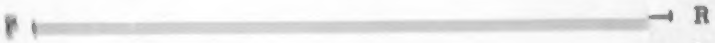
c)



d)



e)



7. Draw a segment 8 cm. in length.

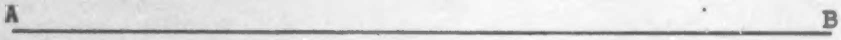
8. Draw a segment 2 in. in length.

## Exercise Set 8

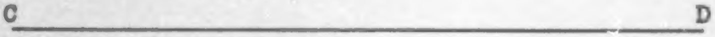
1. Find the length, to the nearest inch, of one of your pencils.

2. Find the length of the same pencil in centimeters.

3. What is the length of $\overline{AB}$, in inches? What is its length in centimeters?



4. $\overline{CD}$ is 4 inches long. What is its length in centimeters?



5. Draw a segment that has a length of 5 inches.

6. Draw a segment whose measure, in inches, is 4.

7. Draw a segment whose measure, in centimeters, is 10.

8. Draw a segment that has a length of 14 centimeters.

9. Copy the sentence and write the words inches or centimeters in the blanks so as to make the sentence true.

A segment which is 5 ________ in length is longer than a segment which is 5 ____________ in length.

## OTHER STANDARD UNITS

### Exploration

The inch unit and the centimeter unit are too small to be used for measuring long distances. As you know, larger units are used for this purpose. Three which you know are named the foot (ft.), the yard (yd.), and the mile (mi.).

Recall what you know about these unit segments.

1. Draw a segment 1 foot long. Draw a segment 1 yard long.

2. Draw other segments 2 ft., 3 ft., and 4 ft. long.

3. Measure each of the segments in inches. List your results like this:

| Length in feet | Length in inches |
|---|---|
| 1 ft. | _____ inches |
| 2 ft. | _____ inches |
| 3 ft. | _____ inches |
| 4 ft. | _____ inches |

Which of these segments was one yard in length?

4. Draw a line segment 18 inches long. Name it $\overline{AB}$.

Mark a point C on $\overline{AB}$ so that $\overline{AC}$ is 1 ft. long.

How long is $\overline{CB}$?

The length of $\overline{AB}$ is 1 ft. _____ in.

5. Draw a line segment 29 in. long. Name it $\overline{AB}$.
Find a point C on $\overline{AB}$ so that $\overline{AC}$ is 1 ft. long.
Then find a point D on $\overline{CB}$ so that $\overline{CD}$ is 1 ft. long.

How long is $\overline{AD}$?
How long is $\overline{DB}$?
$\overline{AB}$ is ____ ft. ____ in. long.

6. Draw segments with these lengths in inches.
Then tell the length in feet and inches.

| Segment | Length in inches | Length in feet and inches |
|---|---|---|
| $\overline{EF}$ | 15 in. | ____________ |
| $\overline{GH}$ | 20 in. | ____________ |
| $\overline{IJ}$ | 27 in. | ____________ |
| $\overline{KL}$ | 30 in. | ____________ |
| $\overline{MP}$ | 36 in. | ____________ |

7. On $\overleftrightarrow{m}$ mark of $\overline{AB}$ whose length is 1 foot. Then mark off $\overline{BC}$ whose length is 3 inches so that C is not on $\overline{AB}$. What is the length of $\overline{AC}$ in feet and inches? What is the length of $\overline{AC}$ in inches?

8. On $\overleftrightarrow{k}$ mark off $\overline{RS}$ whose length is 2 feet. Then mark off $\overline{ST}$ whose length is 7 inches, so that T is not on $\overline{RS}$. What is the length of $\overline{RT}$ in feet and inches? What is the length of $\overline{RT}$ in inches?

9. Draw segments of the lengths given. Then tell the length in another way.

| Segment | Length in feet and inches | Length in inches |
|---|---|---|
| $\overline{AB}$ | 2 ft. 1 in. | ________ |
| $\overline{CD}$ | ____________ | 21 in. |
| $\overline{EF}$ | 1 ft. 7 in. | ________ |
| $\overline{GH}$ | ____________ | 32 in. |

BRAINTWISTERS

1. A mile is a distance of 5,280 feet. About how many steps would you take to walk a mile?

2. If a car is traveling 60 miles an hour, how far will it travel in one minute?
How long does it take you to walk a mile?

3. Suppose someone ran a mile in 238 seconds. How many minutes and seconds did he take?

4. Suppose the four men in a mile relay event ran their "quarter" mile in the times of 46 seconds, 47 seconds, 47 seconds, and 45 seconds. What was the time for the relay team to run the mile?

## Exercise Set 9

1. If segments have these lengths, which is longer?

a) 10 inches or 1 foot? ________

b) 4 feet or 1 yard? ________

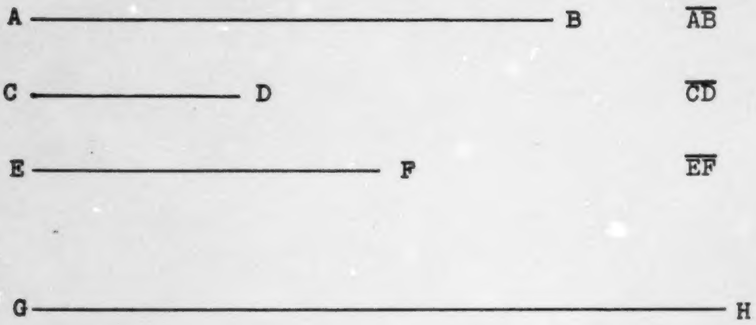
c) 1 inch or 1 centimeter? ________

d) 1 foot or 10 centimeters? ________

e) 1 ft. 7 in. or 2 ft.? ________

f) 28 inches or 2 feet? ________

2. Look at each segment and its measure, then write the unit.



| Segment | Length |
|---|---|
| $\overline{AB}$ | 3______ |
| $\overline{CD}$ | 3______ |
| $\overline{EF}$ | 5______ |
| $\overline{GH}$ | 4______ |

3. Complete:

| | Length in feet and inches | Length in inches |
|---|---|---|
| $\overline{XY}$ | 3 ft. 1 in. | ________ |
| $\overline{MN}$ | ____________ | 30 in. |
| $\overline{DE}$ | 1 ft. 10 in. | ________ |
| $\overline{FG}$ | ____________ | 44 in. |

4. Write the name of the unit which makes the sentence reasonable. Use centimeters, inches, feet, yards, or miles.

a) The seat of the teacher's chair is 16 ______ above the floor.

b) Helen is in the fourth grade. Her height is 130 ________.

c) The lake is 4 ________ long.

d) The height of a tall man is 2 ________.

e) A crayon is about 10 __________ long.

f) It is 2 __________ from Bob's house to the park.

5. If segments have these lengths, which is longer? How much longer?

| | Which is Longer? | How much Longer? |
|---|---|---|
| a) 23 inches or 1 foot | ______ | ______ |
| b) 18 inches or 2 feet | ______ | ______ |
| c) 4 feet or 1 yard | ______ | ______ |
| d) 1 ft. 8 in. or 16 in. | ______ | ______ |
| e) 1 yd. 4 in. or 42 in. | ______ | ______ |
| f) 1 yd., 2 ft., or 7 ft. | ______ | ______ |
| g) 1 mile or 3,495 feet | ______ | ______ |

6. You know that 1 mile is 5,280 feet. How many yards is that?

7. A Boy Scout can walk a mile in 12 minutes if he uses the Boy Scout pace. How many miles can he go in one hour?

# COMBINING LENGTHS

## Learning to Combine Lengths

### Exploration

| Pupil | Height |
|---|---|
| Bill | 4 ft. 8 |
| Betty | 4 ft. 2 |
| Jim | 5 ft. |
| Helen | 4 ft. 6 |

1. The teacher made a record of the heights of children in his fourth grade class.

The children knew that he meant Bill's height was 4 feet and 8 inches. Betty's height was 4 feet and 2 inches. Jim's height was 5 feet and 0 inches. How many feet and inches is Helen's height?

We often use more than one unit to tell sizes.

How would you use more than one unit to measure the door in your classroom?

2. Now we will learn how to combine lengths.

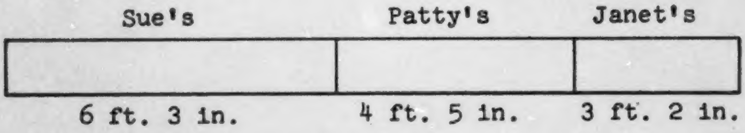
Sue, Patty, and Janet are decorating a room for a party.

They plan to put crepe paper around the windows.

The girls had strips of paper with these lengths:

| | | |
|---|---|---|
| Sue | 6 ft. | 3 in. |
| Patty | 4 ft. | 5 in. |
| Janet | 3 ft. | 2 in. |

a) If they put their strips of paper end to end it would look like this:



To find the length of the combined strips, we change all measures so that we use only one unit, the inch.

How many inches of paper did Sue have? ____

How many inches of paper did Patty have? ____

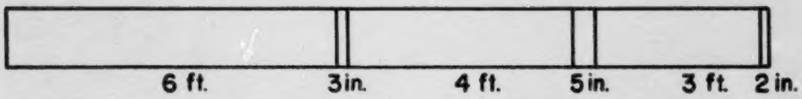
How many inches of paper did Janet have? ____

Now we can add the measures: 75 + 53 + 38 = 166.

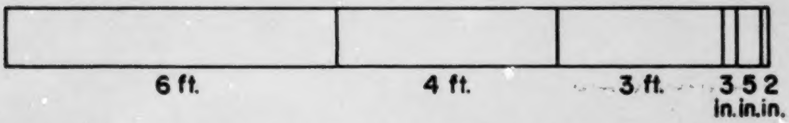
The length of the paper was 166 inches.

Express this length in feet and inches.

b) We can show the combined strips by another sketch like this:



c) Now let us think about the combined strips in a different order. We can show our thinking by this sketch:



The answer could be found by adding the measures which go with the same unit.

6 ft. 3 in.

6 + 4 + 3 = 13 4 ft. 3 + 5 + 2 = 10 5 in.

3 ft. 2 in.

13 ft. 10 in.

Usually we write:

6 ft. 3 in.

4 ft. 5 in. The girls have 13 ft.

3 ft. 2 in. 10 in. of crepe paper.

13 ft. 10 in.

3. Which strip of paper is longer?

Patty's
Janet's
4 ft. 5 in.
3 ft. 2 in.

a) The length of Patty's paper, in inches 53 in.

The length of Janet's paper, in inches 38 in.

The lengths are in the same unit.

We subtract the measures: $53 - 38 = n$

$$\begin{array}{r} 53 \\ -\ 38 \\ \hline 15 \end{array}$$

Patty's strip is 15 inches longer than Janet's.

How many feet and inches is this?

b) We can work the problem another way.

We subtract the measures which go with the same unit.

$4 - 3 = 1$ $\qquad$ $$\begin{array}{ll} 4 \text{ ft.} & 5 \text{ in.} \\ \underline{3 \text{ ft.}} & \underline{2 \text{ in.}} \\ 1 \text{ ft.} & 3 \text{ in.} \end{array}$$ $\qquad$ $5 - 2 = 3$

The work is usually written like this:

$$\begin{array}{ll} 4 \text{ ft.} & 5 \text{ in.} \\ \hline 3 \text{ ft.} & 2 \text{ in.} \end{array}$$

$$\begin{array}{ll} 4 \text{ ft.} & 5 \text{ in.} \\ 3 \text{ ft.} & 2 \text{ in.} \\ \hline 1 \text{ ft.} & 3 \text{ in.} \end{array}$$

The length of Patty's paper is 1 ft. 3 in. more than the length of Janet's paper.

4. Martin, Charles, and John had a contest to see who could throw a football the farthest. See the record of their distances.

| | |
|---|---|
| Martin | 18 yd. 2 ft. |
| Charles | 16 yd. 1 ft. |
| John | 17 yd. 2 ft. |

a) How much farther did Martin throw than Charles?

b) How much farther did Martin throw than John?

c) How many feet did Charles throw the ball?

d) How far was the football thrown in three tries?

Can we add the three measures?

We can add only the measures which have the same unit.

| | |
|---|---|
| 18 | 2 |
| 16 | 1 |
| 17 | 2 |
| 51 | 5 |

The football was thrown 51 _____ 5 _____ by the boys.

5 ft. = _____ yd. _____ ft.

51 yd.
1 yd. 2 ft.
52 yd. 2 ft.

51 yd. 5 ft. is the same measure as 52 yd. 2 ft.

## Exercise Set 10

1. Dick has two pencils. One pencil is 7 inches long and one is 6 inches long. When put end to end his two pencils show a segment which measures ____ ft. ____ in.

2. Jane has a piece of ribbon 2 ft. 8 in. in length; Sara's ribbon is 3 ft. 9 in. long.

   a) How much shorter is Jane's ribbon than Sara's?

   b) The two ribbons, when put end to end, make a piece _____ ft. _____ in. long.

3. Add these measures. (Write the lengths in the answers in two ways, as shown in exercise a).

a) 6 yd. 2 ft.
7 yd. 1 ft.
9 yd. 2 ft.
22 yd. 5 ft. = 23 yd. 2 ft.

b) 23 ft. 8 in.
35 ft. 2 in.
46 ft. 10 in.

c) 38 yd. 2 ft.
23 yd. 2 ft.
66 yd. 2 ft.

d) 11 yd. 2 ft.
37 yd. 1 ft.
9 yd. 2 ft.

e) 24 ft. 2 in.
75 ft. 6 in.
67 ft. 6 in.

f) 8 in.
5 in.
4 in.

4. Subtract these measures.

a) 5 yd. 2 ft.
2 yd. 1 ft.

c) 33 yd. 2 ft.
8 yd. 2 ft.

b) 37 ft. 11 in.
18 ft. 8 in.

d) 26 ft. 9 in.
9 ft. 4 in.

5. Here are the names and heights (in inches) of 30 pupils of one classroom.

COLUMNS

ROWS

| | | | | |
|---|---|---|---|---|
| John<br>56 | Mary<br>54 | Jeannie<br>53 | Royce<br>63 | Tom<br>52 |
| Denise<br>53 | Paul<br>61 | Jack<br>63 | Phyllis<br>52 | Helen<br>52 |
| Fran<br>55 | Fred<br>59 | Martha<br>54 | Walter<br>61 | Viola<br>60 |
| Bill<br>52 | Glen<br>52 | Lenore<br>61 | Gary<br>56 | Boyd<br>65 |
| Sarah<br>52 | Greg<br>52 | Valaree<br>63 | Jake<br>56 | Gerry<br>65 |
| Edna<br>52 | Will<br>54 | James<br>52 | Hill<br>65 | Jo<br>64 |

a) Which row of pupils has the greatest total height? How much is this greatest total height?

b) Which column of pupils has the greatest total height? How much is this greatest total height?

c) Is there one pupil who is sitting both in the row and in the column of greatest total height? Who?

6. Use the table of exercise 5 to answer the following.

a. Which column of pupils has the least total height?

How much is the least total height?

b. Which row of pupils has the least total height?

How much is this total height?

c. Is there one pupil who is sitting both in the row and in the column of least total height? Who?

## PERIMETER

### Lengths of Simple Closed Curves

### Exploration

1. Take a piece of wire, 15 inches long.

   Bend it to make a simple closed curve, like this:

   What is the length of the wire when it is bent in the shape of this curve?

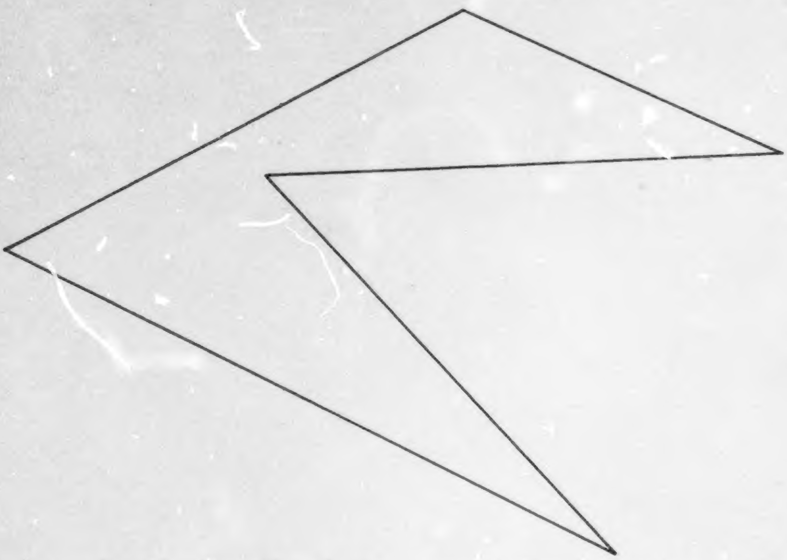
2. Straighten out the wire, and bend it to form a different closed curve.

   What is the length of the new curve?

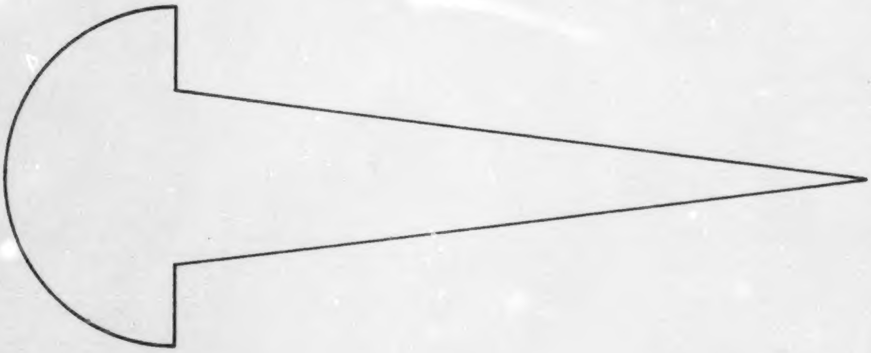
   What happens to the length of the wire when you change the shape of the simple closed curve?

3. Straighten out your wire.

Can you use it to find the length of the curve drawn below?



4. Is the length of the curve below, greater or less than 15 inches?



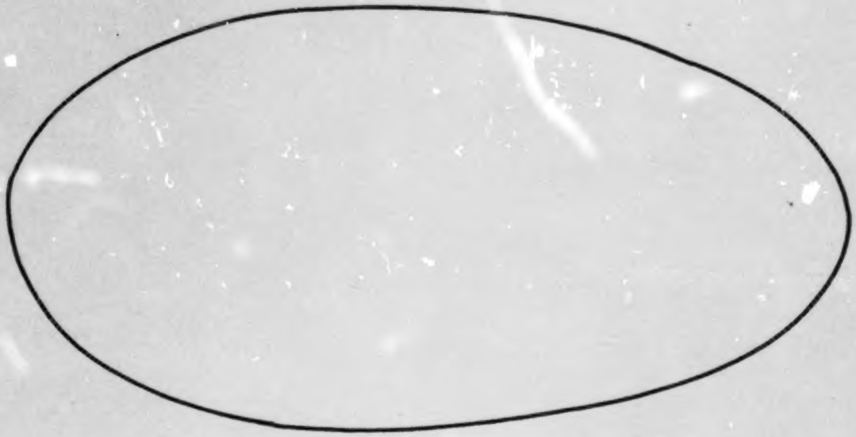
How much less than 15 inches?

Did you use your ruler to measure the length of the wire that remained after you bent the piece of wire to fit the curve above?

5. Take another piece of wire whose length you do not know.

Bend it to fit the curve below.

If there is wire left over, bend it back and out of the way.



Can you think of some way of using the wire outline to find the length of the curve?

If you straighten the wire out, will the length of the part of the wire that outlined the curve change?

Straighten out the wire.

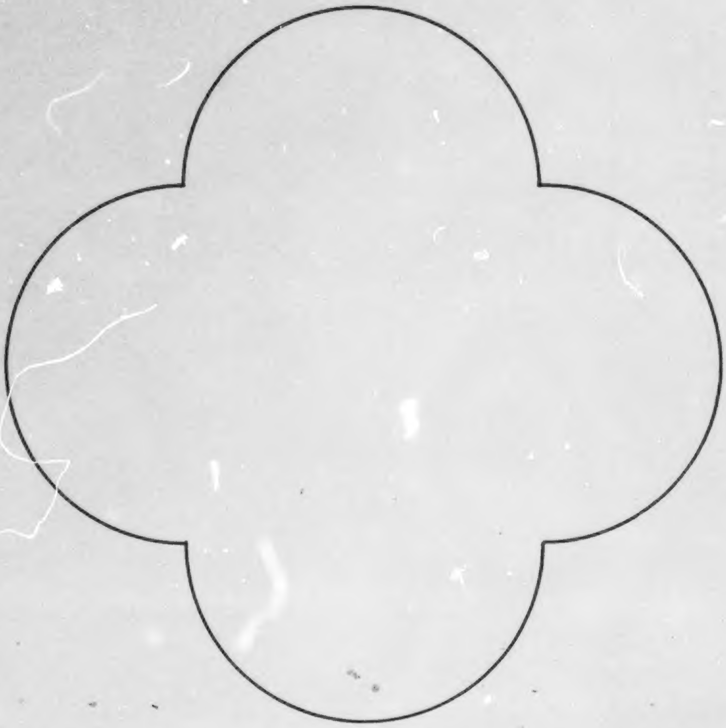
Measure it with a ruler.

What is the length of the curve?

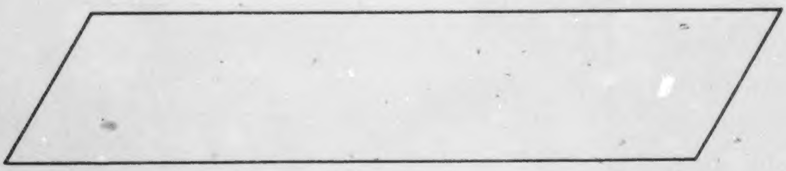
## Exercise Set 11

Use pieces of wire and a ruler to find the lengths of the curves.

1.



2.



Remember that this simple closed curve is called a polygon because it is the union of line segments.

3. Take a piece of wire. Bend it, using the whole piece of wire, so that you make an equilateral triangle.

What is the length of the triangle?

4. Bend a piece of wire so that, using the whole piece of wire, you have a polygon with four sides of the same length.

Find the length of the polygon.

5. Use the same piece of wire as in exercise 4 and make a different polygon with four sides of the same length. Without straightening the wire, do you know the length of this polygon?

6. Find a model of a circle in your home.

Use a piece of wire or a piece of string to help you find the length of the circle.

Did you need a ruler?

7. Cut a model of a triangle with its interior from a piece of cardboard.

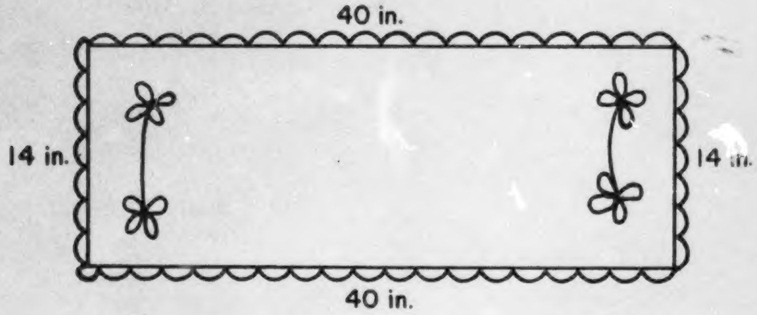
Can you find the length of the triangle?

Did you use a piece of wire?

Could you have found the length of the triangle using the ruler only?

PERIMETERS OF POLYGONS

## Exploration



Joan wished to buy some lace edging to trim a scarf.

The scarf was 40 inches long and 14 inches wide.

How much edging does Joan need?

The number sentence which tells the measure of the length of the edging Joan needs to trim all four edges of the scarf is:

$$40 + 14 + 40 + 14 = 108$$

The length of the edging that Joan must buy is 108 inches.

How many yards of edging does she need?

The perimeter of the rectangle in this example is 108 inches.

You have found the length of many simple closed curves.

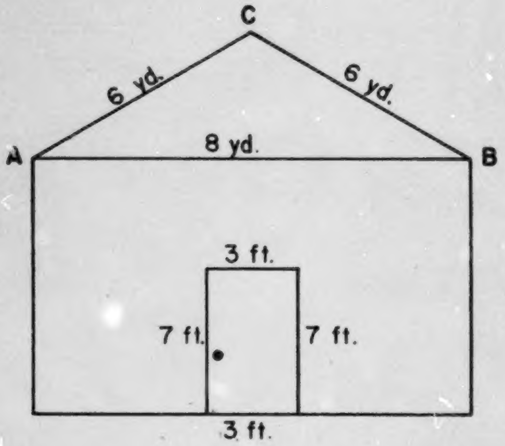
When the curve is a polygon whose sides are line segments, we call the length of the polygon its perimeter.

The perimeter gives a number and a unit of measure.

2. The Jones family decided to decorate the front of their home for the Christmas season.

Johnny wanted to put a string of colored lights on the house along the triangle ABC. Mary wanted to put a string of colored lights around the doorway.

Mr. Jones said he would buy lights for the door or the roof. He would not buy lights for both. Also, he would decorate the one which required the shorter string of lights.



Johnny measured the 3 sides of the triangle.

Mary measured the 4 sides of the rectangle around the door.

Each reported that the sum of the measures of the sides of the figure he measured was 20.

Johnny said: "6 + 6 + 8 = 20"

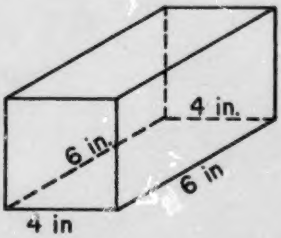
Mary said: "3 + 7 + 3 + 7 = 20"

What other fact did Mr. Jones need to know before he could make a decision about which part of the house to decorate?

Did the Jones family decorate the door or the roof?

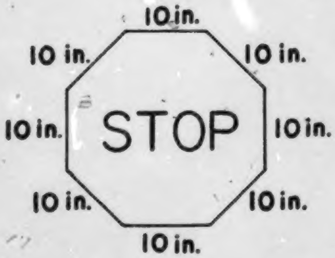
## Exercise Set 12

1. Joe made a cardboard model of a chalkbox. He wished to tape the edges of the bottom of the box with scotch tape. How much scotch tape did he need? (There is no overlap at the corners.)



Do the edges of the bottom make a rectangle?
Did you find the sum of the measures of the sides of the rectangle, or did you find the perimeter of the rectangle?

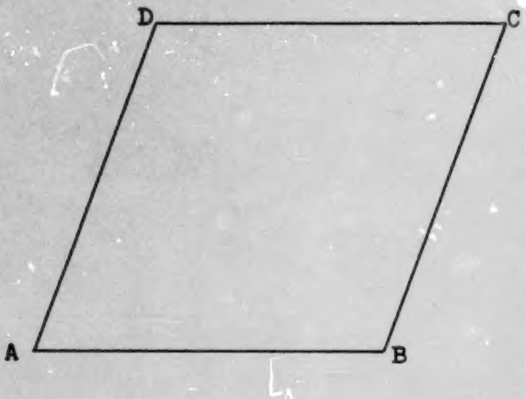
2. The police department of a town is painting a thin black border around the edge of the STOP signs. How many inches of border must be painted on each sign?



The edge of the STOP sign represents a polygon.
What is the perimeter of this polygon?

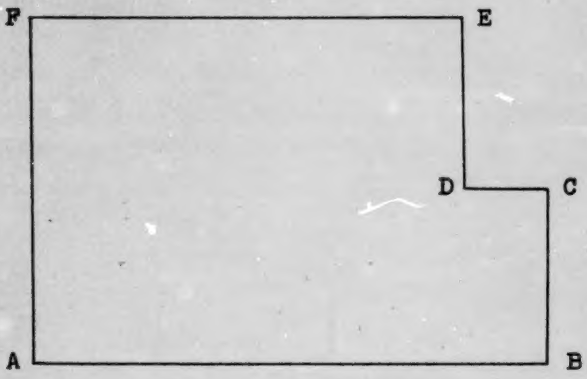
3. Use your ruler to find, to the nearest inch, the measure you need to find the perimeter of each polygon.

a)



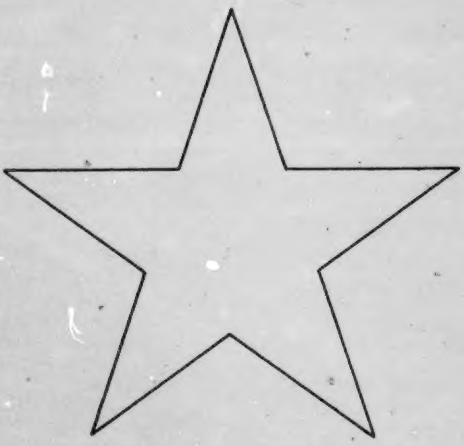
The perimeter of figure ABCD is ________.

b)



The perimeter of figure ABCDEF is ________.

c)



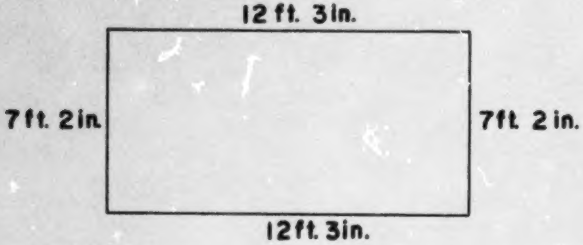
The perimeter of the star is ________.

## FINDING PERIMETERS

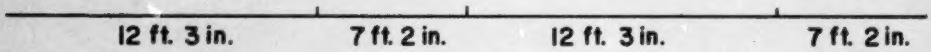
### Exploration

Here is a plan for the floor of a rectangular room.

What is the perimeter of the edge of the floor?

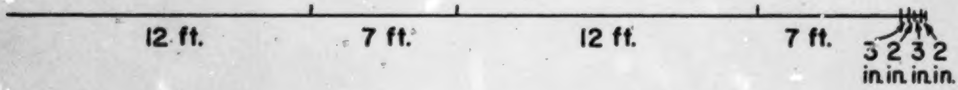

12 ft. 3 in.
7 ft. 2 in.
7 ft. 2 in.
12 ft. 3 in.

If we place a piece of wire around the edge of this floor plan, then straighten out the wire, we can picture the wire like this:


12 ft. 3 in.
7 ft. 2 in.
12 ft. 3 in.
7 ft. 2 in.

Imagine that we cut the wire into eight pieces.

We put them together again, as shown in this picture.


12 ft.
7 ft.
12 ft.
7 ft.
3 in.
2 in.
3 in.
2 in.

Has the length changed?

We add the measures which have the same unit.

$$12 + 7 + 12 + 7 = 38$$

$$3 + 2 + 3 + 2 = 10$$

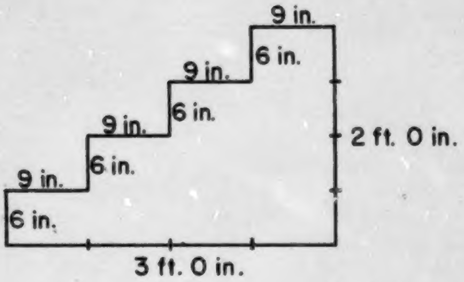
The length, or the perimeter, of the rectangle is 38 ft. 10 in. Remember we add the measures. We do not add the units. We add only those measures that were made using the same unit.

## Exercise Set 13

1. Find the perimeter of a corner piece of land in the shape of a triangle if the lengths of its sides are 50 feet 4 inches, 80 feet 7 inches, and 50 feet 4 inches.

2. A yardstick is 1 inch wide. Find the perimeter of the face of the yardstick that shows the scale. Give your answer in yards and inches. Give your answer again in feet and inches.

3. Find the perimeter of the polygon pictured below:



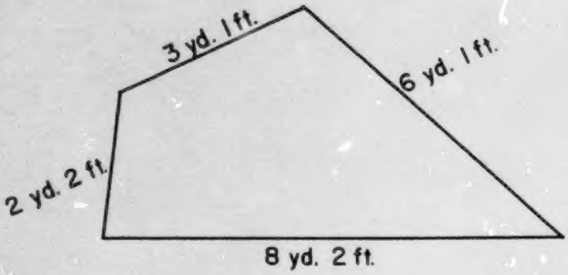
a) Can your answer be written using only one unit?
Write your answer in inches only. in feet only.

b) Can your answer be written using yards only and what we have learned so far?

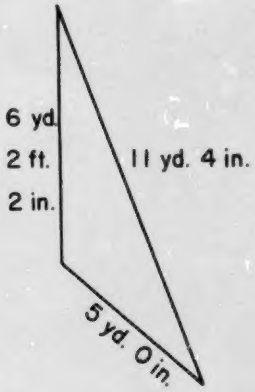
4. In France, a baseball diamond is a square, each of whose sides is 27 meters 70 cm. long. Pierre hits a home run. What is the length of the shortest path he can take, if he touches each base on his way back to home place? You will need to know that 100 centimeters equals 1 meter.

5. Which curve is longer? How much longer?

a)



b)



BRAINTWISTERS

6. A man wants to put a fence all around his land. He knows that the boundary of his land can be thought of as a square. He measures one side and finds that it is between 125 and 126 ft. long. How much fencing should he buy?

7. In Jim's house a piece of glass in a window was broken. His father measured the frame where the glass went. He found that it was a rectangle with 14 inch and 24 inch sides, to the nearest inch. He bought a new piece of glass 14 inches wide and 24 inches long. When he went to put it in the frame he found that it was too long to fit. Can you give a possible reason?

# Chapter 10

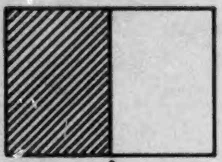
# CONCEPT OF RATIONAL NUMBERS

## IDEA OF RATIONAL NUMBERS

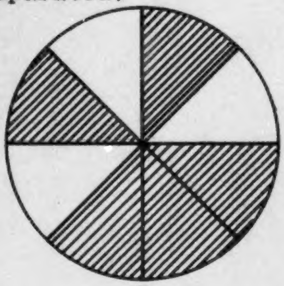
### Exploration

Look at each of the figures on this page.

For each figure, choose a pair of numbers at the right which can be used to talk about the number of parts that are shaded and the number of congruent parts into which each unit region, unit segment, or set has been separated.



A



B

Pairs of Numbers

a. 1 and 4

b. 3 and 4

c. 3 and 5

d. 1 and 2

e. 5 and 8

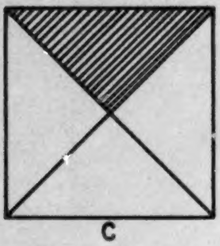
f. 1 and 3

g. 2 and 3

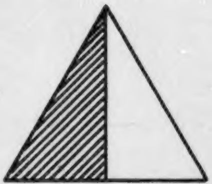
h. 2 and 2

i. 6 and 8

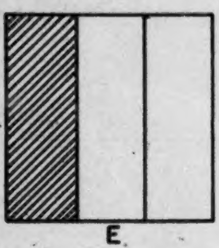
j. 2 and 5



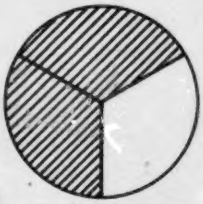
C



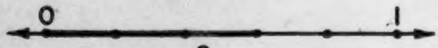
D



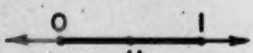
E



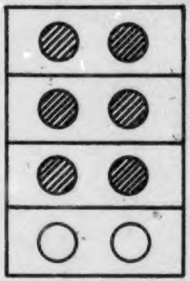
F



G



H



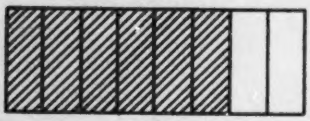
I

Were you able to find a pair of numbers for each? Did you find these -- A-d; B-e; C-a; D-d; E-f; F-g; G-c; H-h; and I-b?

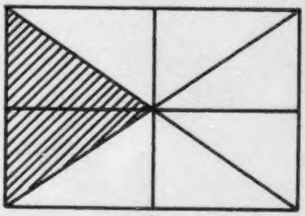
## Exercise Set 1

1. Copy the table and complete it, using the figures A, B, C, D, E, and F.

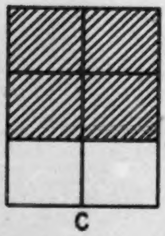
| Figure | Parts Shaded | Congruent Parts in Unit |
|---|---|---|
| A | | |
| B | | |
| C | | |
| D | | |
| E | | |
| F | | |



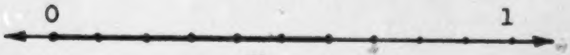
A



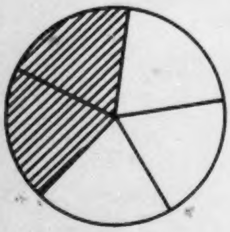
B



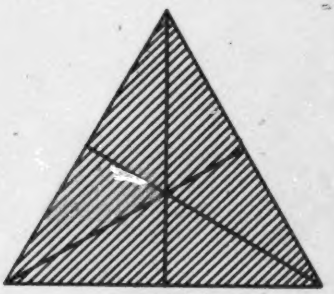
C



D

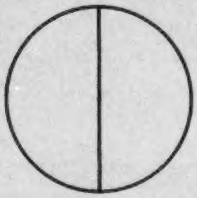


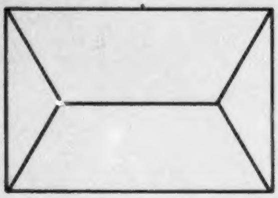
E

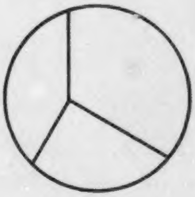


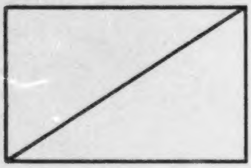
F

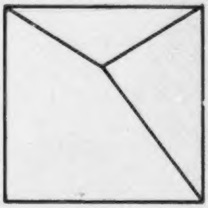
2. Write on your paper the letters from A to G. After each, write Yes if the figure has been partitioned into congruent regions. Write No if the figure has not been partitioned into congruent regions.

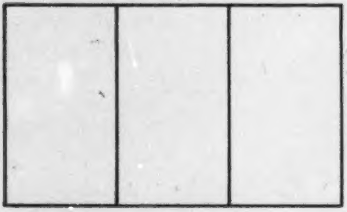

A

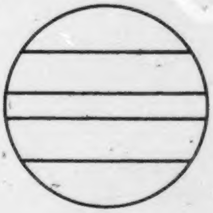

B


C

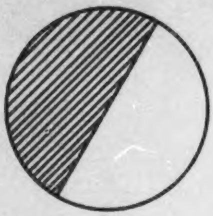

D

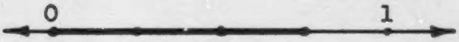

E


F

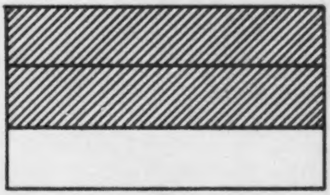

G

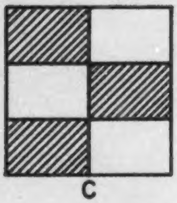
3. What pair of numbers can be used to talk about the shaded region in each figure? Remember we will let the first number of the pair tell how many parts are shaded. We will let the second number of the pair tell into how many congruent parts the unit region has been partitioned.

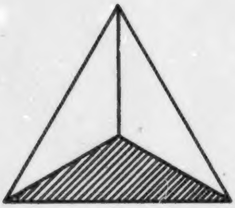

A

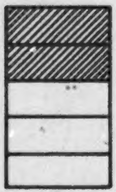


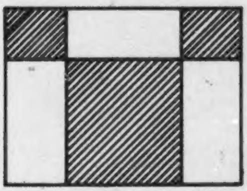
B

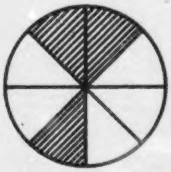

D

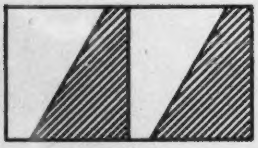

C

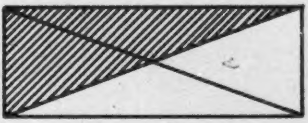

E


G


F


H


I

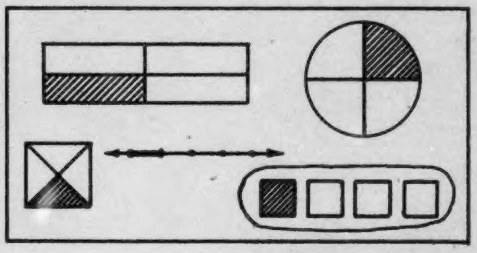

J

Did you find any figures that had not been partitioned into congruent regions? Which ones were they?

## A NEW KIND OF NUMBER

### Exploration

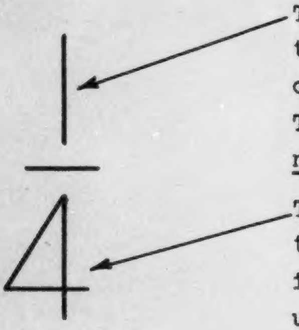
When a region is partitioned into congruent parts and some of these parts are shaded, we use a new kind of number to describe what we see. These new numbers are called rational numbers. $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{3}{8}$ are rational numbers. They are read, "one-half," "one-fourth," and "three-eighths."



Each of these figures at the right suggests the same rational number. The rational number is one-fourth. The symbol, $\frac{1}{4}$, which names the rational number one-fourth is called a fraction. Fractions are written using two numerals. The two numerals are separated by a horizontal bar.

For example:

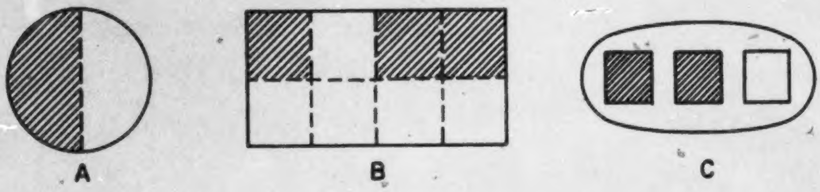
The numerals are 1 and 4.



The numeral above the bar tells the number of congruent parts of equivalent subsets described. The number is called the numerator.

The numeral below the bar tells the number of congruent parts into which the set of objects, unit region, or unit segment has been partitioned. The number is called the denominator.

What rational number is suggested by each of these figures below?



What rational number does each of these figures suggest?

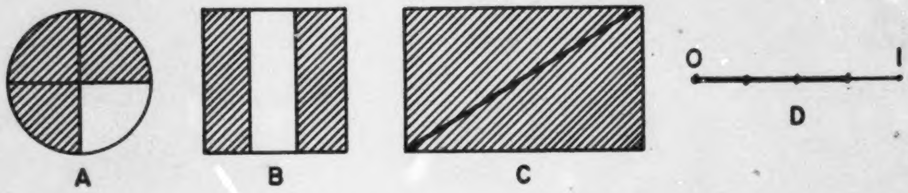


Figure A suggests the rational number, $\frac{3}{4}$, read three-fourths.

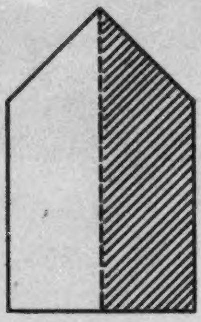
Figure B suggests the rational number, $\frac{2}{3}$, read two-thirds.

Figure C suggests the rational number, $\frac{2}{2}$, read two halves.

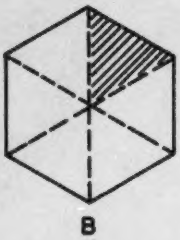
Figure D also suggests the rational number, $\frac{3}{4}$.

## Exercise Set 2

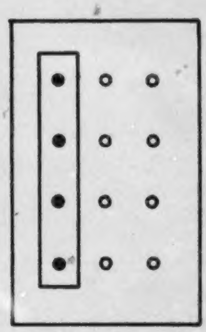
1. For each figure, write a fraction which names the rational number suggested by the shaded region.



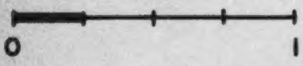
A



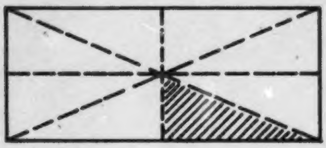
B



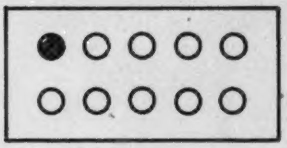
C



D



E



F

2. Write as fractions:

a) one half _____

b) one-third _____

c) one-tenth _____

d) one-eighth _____

e) one-sixth _____

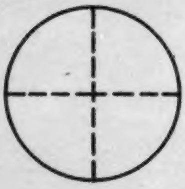
f) one-fourth _____

3. Copy the unit square in Figure H at least six times. (Make more copies if you want them.) In how many ways can you separate the unit square to show:

$\frac{1}{2}$? $\frac{1}{4}$? $\frac{1}{8}$?

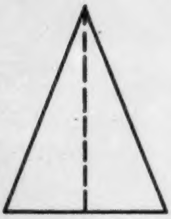
H

4. Copy and shade the part which is described by the fraction below each figure.



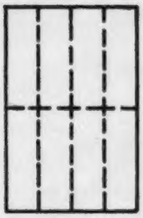
A

$\frac{1}{4}$



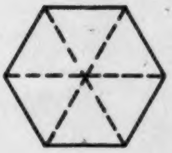
B

$\frac{1}{2}$



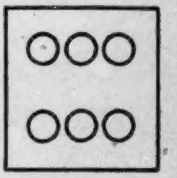
C

$\frac{1}{8}$



D

$\frac{1}{6}$



E

$\frac{1}{3}$

5. Copy and complete this chart.

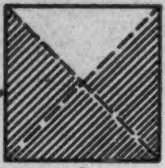
| Fraction | Number of Congruent Parts in Unit | Number of Congruent Parts Counted |
|---|---|---|
| $\frac{1}{4}$ | | |
| $\frac{1}{2}$ | | |
| $\frac{1}{10}$ | | |
| $\frac{1}{6}$ | | |
| $\frac{1}{3}$ | | |
| $\frac{1}{8}$ | | |

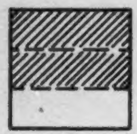
6. On your paper, make 6 copies of the unit region shown below. Make the unit regions the same size. Then show a picture that suggests each of the rational numbers named in exercise 5.

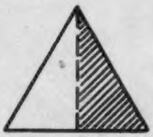
## Exercise Set 3

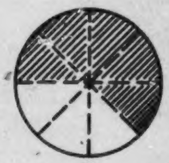
1. Use these figures to complete the chart below.

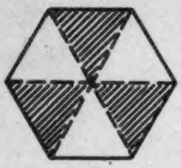
A has been done for you.

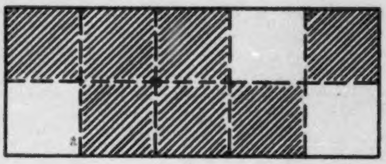

A


B


C


D


E


F

| Figure | Number of Congruent Parts in Figure | Number of Shaded Parts | Rational Number Suggested by Shaded Part |
|---|---|---|---|
| A | 4 | 3 | $\frac{3}{4}$ |
| B | | | |
| C | | | |
| D | | | |
| E | | | |
| F | | | |

2. Using figures A, B, C, D, E, and F of exercise 1, write the name of the rational number suggested by the unshaded part of each figure.

3. Use these figures to complete the sentences below.

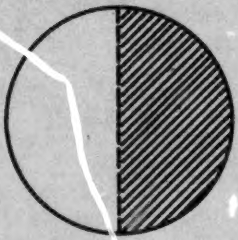


Fig. A

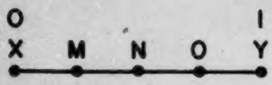


Fig. B

Set C = {1,2,4,6,8,10,12,14}

Fig. C

a) Figure A has been separated into ________ congruent regions. _______ region has been shaded. The shaded region is best described by the rational number named by the fraction ________.

b) Points M, N, and O separate $\overline{XY}$ into _____ congruent segments. m $\overline{XM}$ = ______.

c) Set C has _______ members. _____ member names an odd number. This member is ______ of all the members of Set C.

4. Study your answers to exercises 1, 2, and 3. Copy and then write "above" or "below" in each blank.

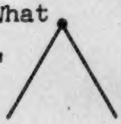
a) The numeral ______ the bar names the number of congruent parts into which the unit has been separated.

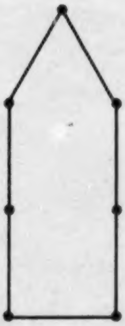
b) The numeral ______ the bar names the number of congruent parts which are described.

5. Ann watched 3 television programs. Each was $\frac{1}{4}$ of an hour long.

a) How long did Ann watch television?

b) How much longer would she need to watch TV to make her total time 1 hour?

6. A figure like the one pictured below was made by laying toothpicks, each the same size, end-to-end. What fractional part of the perimeter is the "roof"  ?

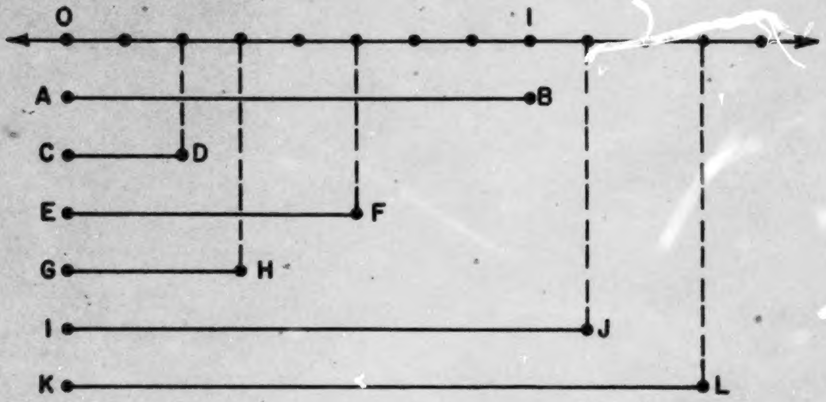


I.

## RATIONAL NUMBERS GREATER THAN ONE

### Exploration

In the picture below, the line segment AB is 1 unit long.



1. (a) On the number line the unit segment is separable into _____ congruent segments.

(b) Use a fraction: Each small segment is _____ of the unit segment.

(c) The measure of $\overline{AB}$ is 1. The measure of $\overline{AB}$ is also _____. (Use a fraction.)

(d) Is $\frac{8}{8}$ the measure of line segment AB?

(e) Is $\frac{2}{8}$ the measure of line segment CD?

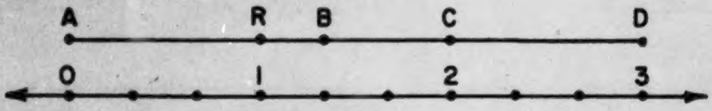
(f) Is $\frac{5}{8}$ the measure of line segment EF?

(g) Is $\frac{3}{8}$ the measure of line segment GH?

(h) Is $\frac{9}{8}$ the measure of line segment IJ?

(i) Is $\frac{11}{8}$ the measure of line segment KL?

2. Each unit segment of the number line below has been separated into 3 congruent segments. $\overline{AR}$ is the same length as the unit segment.

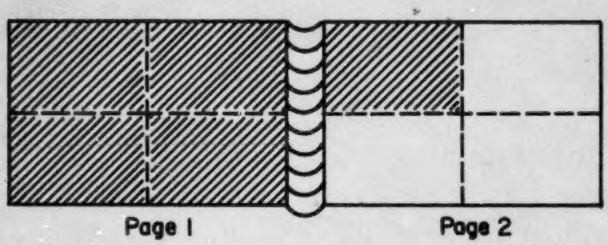


Use this number line to answer the questions.

(a) What fraction names the measure of $\overline{AR}$?

(b) What fraction names the measure of $\overline{AB}$? $\overline{AC}$? $\overline{AD}$?

3. Bill has a photograph album. Each page is separated into 4 congruent parts. On each page he can place 4 pictures.



If Bill pastes 5 pictures in his album, he will cover $\frac{4}{4}$ of one page and $\frac{1}{4}$ of another page. What rational number describes the number of pages covered?

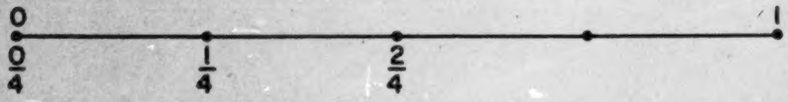
Fractions like $\frac{2}{8}$, $\frac{5}{8}$, $\frac{3}{8}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{4}{5}$ tell us that the measure of a segment or a region is less than 1.

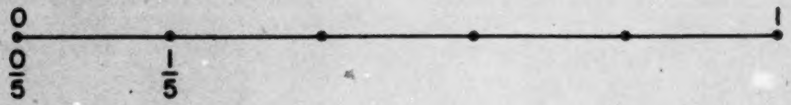
Fractions like $\frac{8}{8}$, $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$, $\frac{5}{5}$, $\frac{6}{6}$ tell us that the measure of a segment or region is exactly 1.

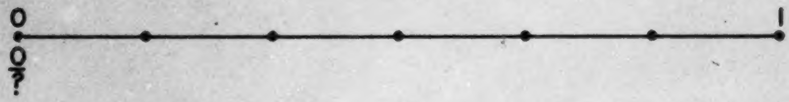
Fractions like $\frac{9}{8}$, $\frac{11}{8}$, $\frac{3}{2}$, $\frac{4}{3}$, $\frac{5}{4}$, $\frac{5}{3}$, $\frac{5}{2}$ tell us that the measure of a segment or region is greater than 1.

## Exercise Set 4

1. Copy the unit segments below. The dots separate each unit segment into smaller, congruent segments. Label each dot correctly.

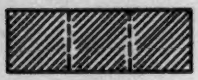


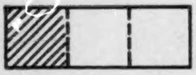


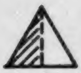


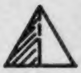
Each of the figures below represents a unit region or unit segment.

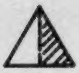
2. Study these diagrams. Then answer the questions on the next page.

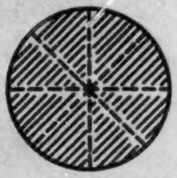

A

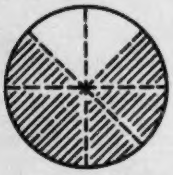

B

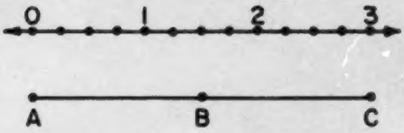

C


D


E


F


G



H

a) How many thirds are there in A?

How many thirds are there in B?

How many thirds are shown in A and B together?

What rational fraction is suggested by the shaded region of A and B together?

What rational number is suggested by the unshaded region of A and B together?

b) What rational number is suggested by the shaded region in C? in D? in E?

What rational number is suggested by the unshaded region in C? in D? in E?

What rational number best describes the shaded regions in C, D, and E altogether?

What rational number best describes the unshaded regions in C, D, and E altogether?

c) What rational number is suggested by the shaded region of F and G together?

What rational number is suggested by the unshaded region of F and G together?

d) In Figure H, what rational number is the measure of $\overline{AB}$? of $\overline{AC}$?

3. For each figure, write the fraction that names the rational number suggested by the shaded part.

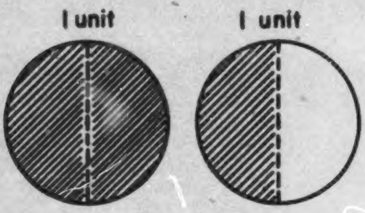

1 unit
1 unit

FIG. A

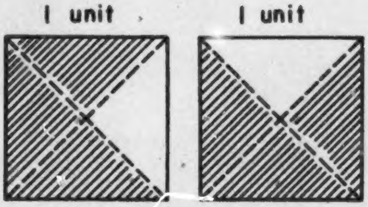

1 unit
1 unit

FIG. B

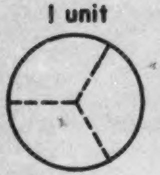


FIG. C

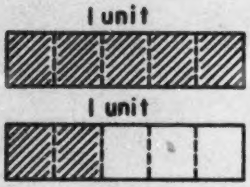

1 unit
1 unit

FIG. D

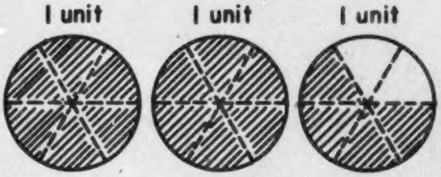
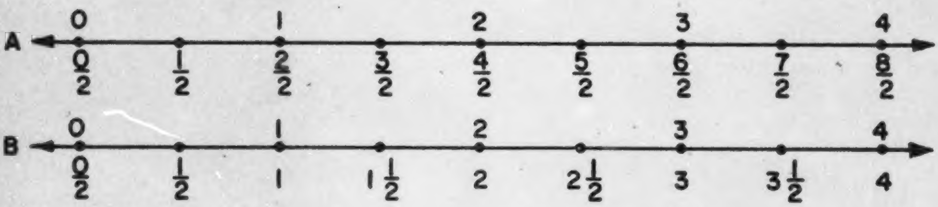

1 unit
1 unit
1 unit

FIG. E

4. Using these number lines, complete the sentences below.


A
0
1
2
3
4
0/2
1/2
2/2
3/2
4/2
5/2
6/2
7/2
8/2
B
0
1
2
3
4
0/2
1/2
1
1 1/2
2
2 1/2
3
3 1/2
4

a) 1 one and 1 half = $\frac{\quad}{2}$ or ____

b) 4 = $\frac{\quad}{2}$

c) 3 ones and 1 half = $\frac{\quad}{2}$ or ____

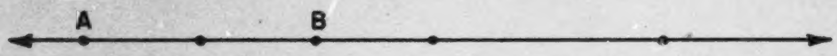
d) 2 ones and 1 half = $\frac{\quad}{2}$ or ____

e) $\frac{3}{2}$ = 1 ____ and 1 ____

f) 2 = $\frac{\quad}{2}$

g) $\frac{2}{2}$ = ____

5. Copy the line segment shown below on your paper.



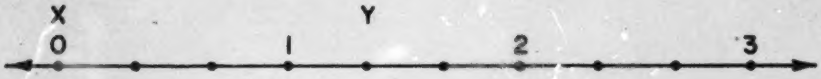
$\overline{AB}$ is a unit segment.

a) Mark a point D so that $\overline{AD}$ is $\frac{1}{2}$ unit long.

b) Mark a point E so that $\overline{AE}$ is $\frac{3}{2}$ units long.

c) Mark a point F so that $\overline{AF}$ is $\frac{5}{2}$ units long.

6. Copy the line segment below. Notice each unit segment has been separated into 3 congruent segments.



Using a certain unit, the measure of $\overline{XY}$ is $\frac{4}{3}$.
Mark new points U, V, and W so that

a) $\overline{XU}$ is 1 unit long.

b) $\overline{XV}$ is $\frac{2}{3}$ unit long.

c) $\overline{XW}$ is $\frac{5}{3}$ units long.

7. Mark is 4 feet tall. What number gives his height in yards?

8. Ellen watched 5 television programs. How many hours did she watch TV if each program was:

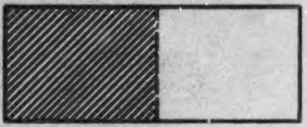
a) $\frac{1}{4}$ of an hour long?

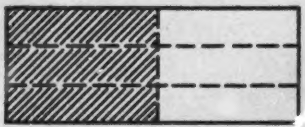
b) $\frac{1}{2}$ of an hour long?

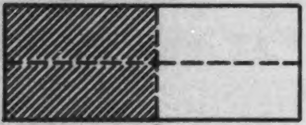
## DIFFERENT NAMES FOR THE SAME NUMBER

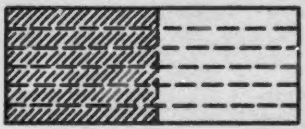
### Exploration

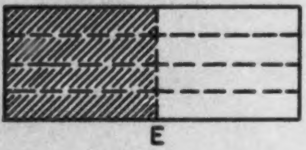
1. The pictures of unit regions below suggest some ways of thinking of one-half.

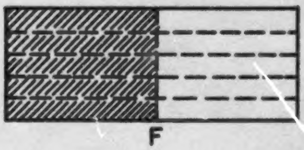

A


B


C


D


E


F

In A, what fraction names the measure of the shaded region?

In B, what fraction names the measure of the shaded region?

In C, what fraction names the measure of the shaded region?

In D, what fraction names the measure of the shaded region?

In E, what fraction names the measure of the shaded region?

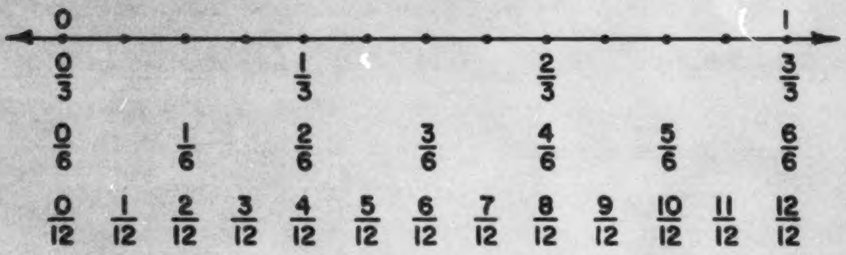
In F, what fraction names the measure of the shaded region?

$\frac{1}{2}$, $\frac{3}{6}$, $\frac{2}{4}$, $\frac{6}{12}$, $\frac{4}{8}$, and $\frac{5}{10}$ are all ways of naming the rational number $\frac{1}{2}$.

We can write: $\frac{1}{2} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10}$.

What are some other fractions that name this same number?

We say that $\frac{1}{2}$ is the simplest name, or simplest form, for this rational number. Can you tell why?


0
1
0/3
1/3
2/3
3/3
0/6
1/6
2/6
3/6
4/6
5/6
6/6
0/12
1/12
2/12
3/12
4/12
5/12
6/12
7/12
8/12
9/12
10/12
11/12
12/12

2. Make true statements by writing a fraction in each blank. Use the number line above to help you.

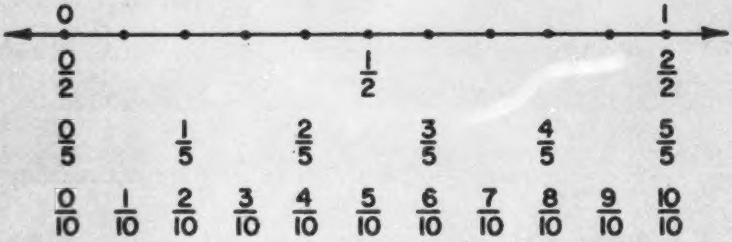
a. $\frac{1}{6}$ = ______

b. $\frac{10}{12}$ = ______

c. $\frac{1}{3}$ = ______ = ______

d. $\frac{4}{6}$ = ______ = ______

e. 1 = ______ = ______ = ______


0
1
0/2
1/2
2/2
0/5
1/5
2/5
3/5
4/5
5/5
0/10
1/10
2/10
3/10
4/10
5/10
6/10
7/10
8/10
9/10
10/10

3. Use the number line above to help you write the missing numerator or denominator.

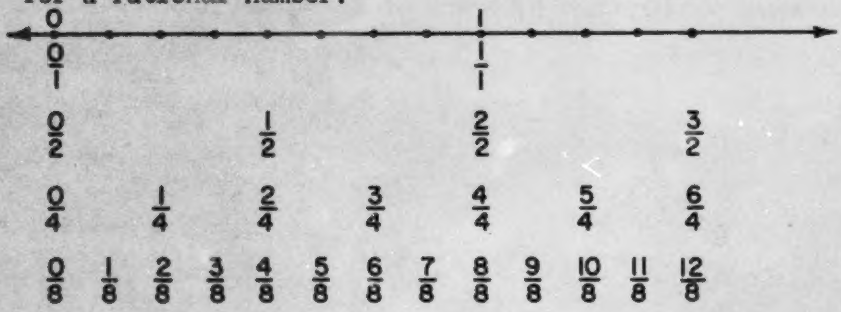
a. $\frac{1}{2} = \frac{n}{10}$

b. $\frac{2}{10} = \frac{1}{n}$

c. $\frac{4}{5} = \frac{n}{10}$

d. $1 = \frac{10}{n} = \frac{8}{5}$

4. Using one number line, we can show many different names for a rational number.



We see that some fractions are names for the same rational number.

What other fractions are names for the rational number $\frac{1}{2}$ ?

What other fraction is a name for the rational number $\frac{1}{4}$ ?

What other fraction is a name for the rational number $\frac{3}{4}$ ?

Can you find other fractions that name the same rational number on this line?

One rational number may be named by many fractions.

The rational number $\frac{1}{4}$ may be named by: $\{\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{6}{24}, \ldots\}$

The rational number $\frac{2}{3}$ may be named by: $\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \ldots\}$

The rational number $\frac{2}{5}$ may be named by: $\{\frac{2}{5}, \frac{4}{10}, \frac{6}{15}, \frac{8}{20}, \ldots\}$

The rational number $\frac{1}{10}$ may be named by: $\{\frac{1}{10}, \frac{2}{20}, \frac{3}{30}, \frac{4}{40}, \ldots\}$

Can you think of other fractions which would name each of these numbers above?

Many fractions can be used to name the same whole numbers.

For example, 1 may be indicated by

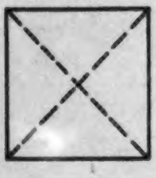
$\frac{2}{2}, \frac{3}{3}, \frac{4}{4}, \frac{5}{5}, \frac{6}{6},$ and so on.

Can you name three other fractions that belong to this set?

## Exercise Set 5

Copy each of these figures.

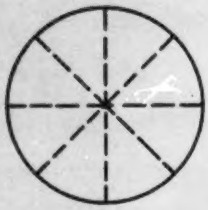
1.



Color $\frac{2}{4}$ of this figure.

$\frac{2}{4}$ is another name for ___.

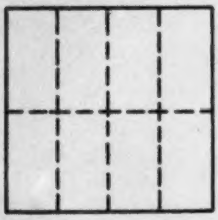
2.



Color $\frac{6}{8}$ of this figure.

$\frac{6}{8}$ is another name for ___.

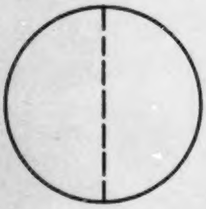
3.



Color $\frac{2}{8}$ of this figure.

$\frac{2}{8}$ is another name for ___.

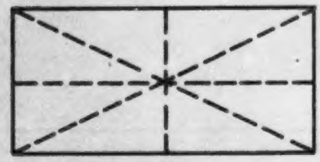
4.



Color $\frac{2}{2}$ of this figure.

$\frac{2}{2}$ is another name for ___.

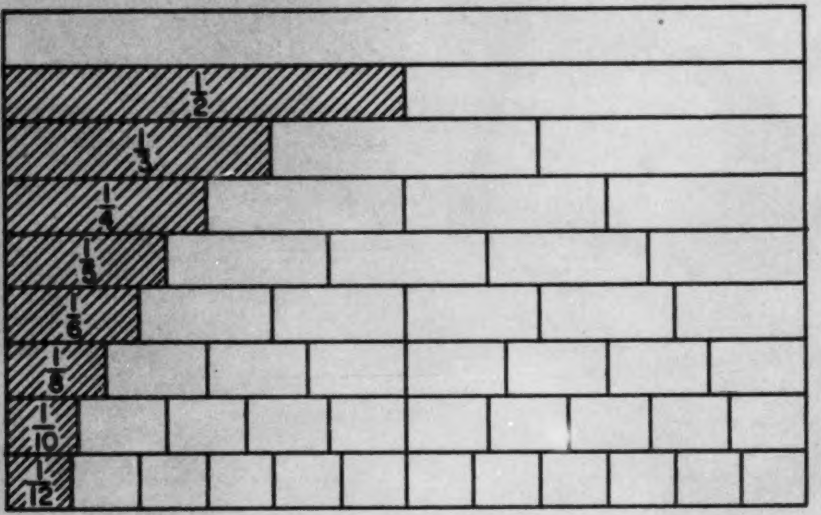
5.



Color $\frac{4}{8}$ of this figure.

$\frac{4}{8}$ is another name for ___.

6. Using this chart, write as many names as you can for


$\frac{1}{2}$
$\frac{1}{3}$
$\frac{1}{4}$
$\frac{1}{5}$
$\frac{1}{6}$
$\frac{1}{8}$
$\frac{1}{10}$
$\frac{1}{12}$

a) $\frac{1}{2}$

b) $\frac{1}{3}$

c) $\frac{1}{4}$

d) $\frac{1}{5}$

e) $\frac{4}{6}$

f) $\frac{6}{8}$

7. Write at least three other fractions which name each of the following rational numbers. If you can write more than three, do so.

a) $\frac{1}{4}$

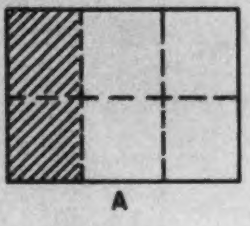
b) $\frac{2}{5}$

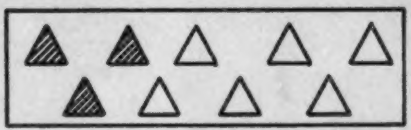
c) $\frac{5}{2}$

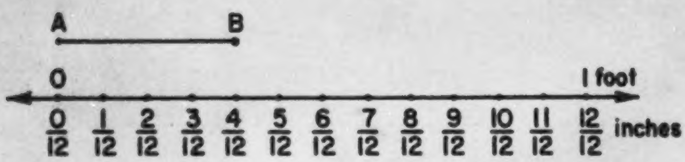
d) $\frac{2}{3}$

e) $\frac{3}{4}$

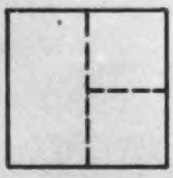
8. The diagrams below suggest three other names for $\frac{1}{3}$. What are they?


A


B


A
B
0
1 foot
0/12
1/12
2/12
3/12
4/12
5/12
6/12
7/12
8/12
9/12
10/12
11/12
12/12
inches

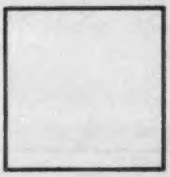
9. Draw 5 boxes like the ones below. Separate each box to show the mathematical sentence written below. The first one is done for you.


$\frac{1}{2} = \frac{2}{4}$

$\frac{4}{8} = \frac{1}{2}$

$\frac{1}{4} = \frac{2}{8}$

$\frac{6}{8} = \frac{3}{4}$


$\frac{2}{6} = \frac{1}{3}$

10. Complete:

a) $1 = \frac{\quad}{2}$

b) $\frac{2}{6} = \frac{\quad}{3}$

c) $\frac{1}{2} = \frac{\quad}{4}$

d) $\frac{1}{4} = \frac{\quad}{8}$

e) $\frac{4}{8} = \frac{\quad}{2}$

f) $\frac{8}{8} = \frac{\quad}{4}$

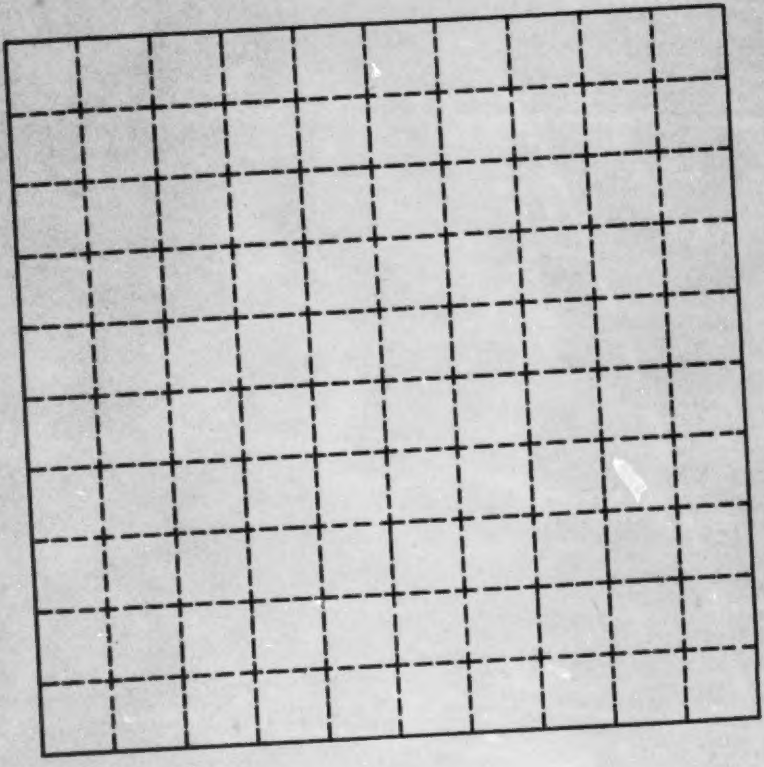
g) $\frac{1}{3} = \frac{\quad}{9}$

h) $\frac{2}{4} = \frac{\quad}{8}$

i) $\frac{6}{8} = \frac{\quad}{4}$

j) $\frac{1}{3} = \frac{\quad}{12}$

11.



The unit square shown on the preceding page has been separated into 100 congruent square regions.

a) Each small square region is what part of the unit square region?

b) Each small square region is what part of 1 row or 1 column of square regions?

c) Each row or each column of square regions is what part of the unit square region?

d) $\frac{4}{10} = \frac{?}{100}$

e) $\frac{7}{10} = \frac{?}{100}$

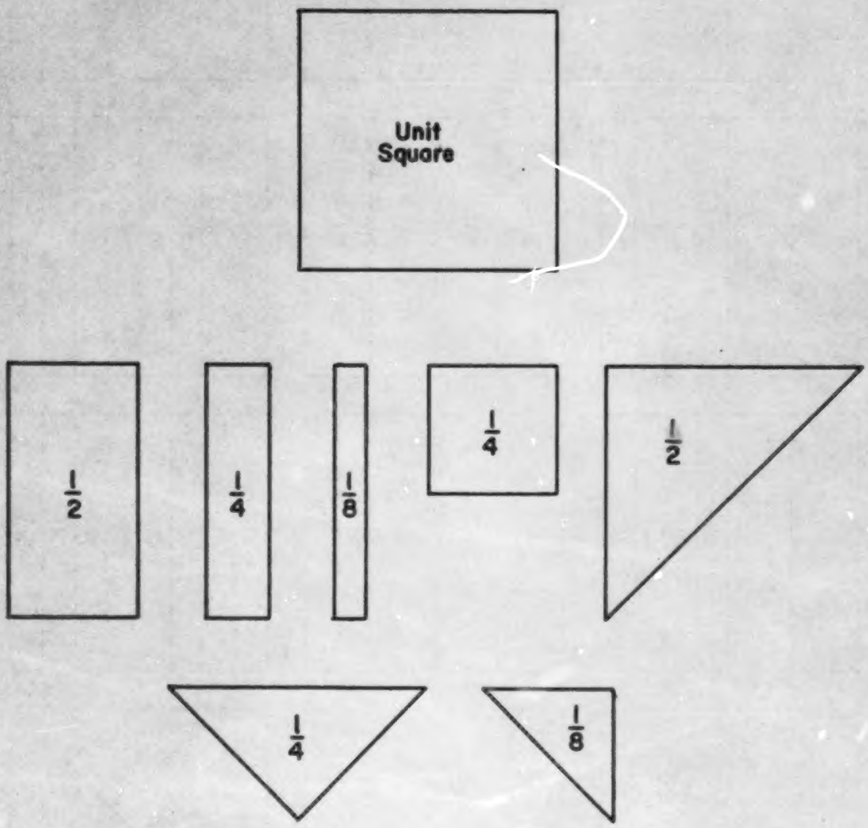
f) $\frac{90}{100} = \frac{?}{10}$

g) $\frac{30}{100} = \frac{?}{10}$

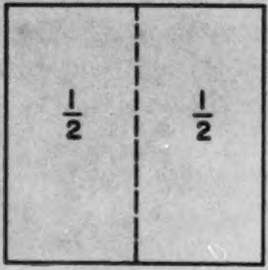
h) How many small square regions should you color if you are to color $\frac{47}{100}$ of the unit square region?

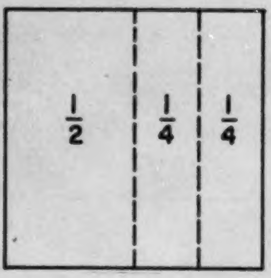
$\frac{83}{100}$ ? $\frac{100}{100}$ ? $\frac{1}{10}$ ? $\frac{7}{10}$ ? and $\frac{10}{10}$ ?

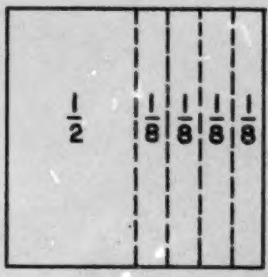
**Puzzle.** In how many different ways can you cover the unit square using the fractional pieces shown? Each piece may be used more than once. You may wish to trace, cut out, and make several copies of each model region before you work your puzzle.

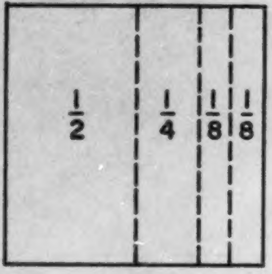

Unit Square
$\frac{1}{2}$
$\frac{1}{4}$
$\frac{1}{8}$
$\frac{1}{4}$
$\frac{1}{2}$
$\frac{1}{4}$
$\frac{1}{8}$

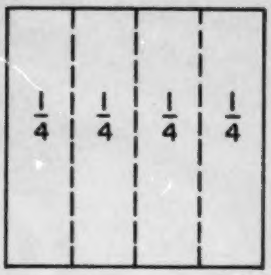
Here are a few solutions to the puzzle on page 541. How many more did you discover?

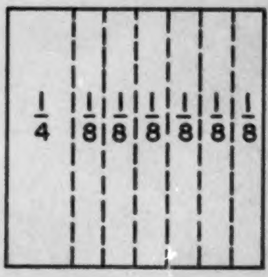

$\frac{1}{2}$
$\frac{1}{2}$

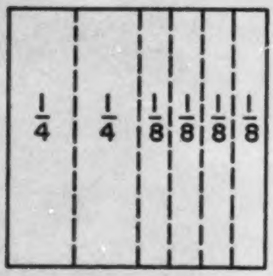

$\frac{1}{2}$
$\frac{1}{4}$
$\frac{1}{4}$

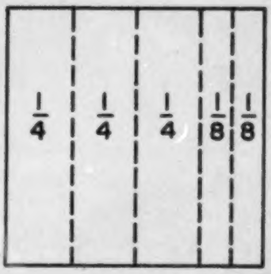

$\frac{1}{2}$
$\frac{1}{8}$
$\frac{1}{8}$
$\frac{1}{8}$
$\frac{1}{8}$

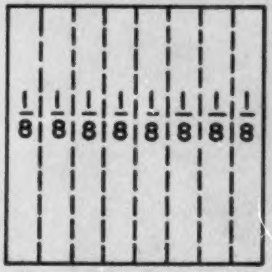

$\frac{1}{2}$
$\frac{1}{4}$
$\frac{1}{8}$
$\frac{1}{8}$

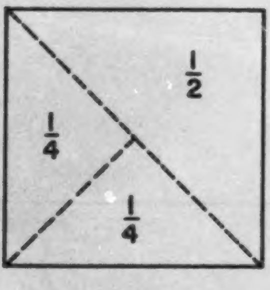

$\frac{1}{4}$
$\frac{1}{4}$
$\frac{1}{4}$
$\frac{1}{4}$

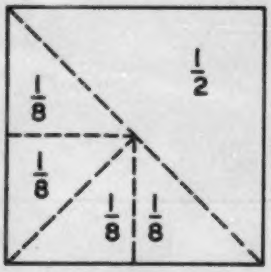

$\frac{1}{4}$
$\frac{1}{8}$
$\frac{1}{8}$
$\frac{1}{8}$
$\frac{1}{8}$
$\frac{1}{8}$
$\frac{1}{8}$

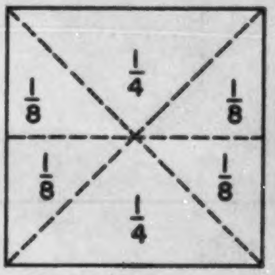

$\frac{1}{4}$
$\frac{1}{4}$
$\frac{1}{8}$
$\frac{1}{8}$
$\frac{1}{8}$
$\frac{1}{8}$


$\frac{1}{4}$
$\frac{1}{4}$
$\frac{1}{4}$
$\frac{1}{8}$
$\frac{1}{8}$


$\frac{1}{8}$
$\frac{1}{8}$
$\frac{1}{8}$
$\frac{1}{8}$
$\frac{1}{8}$
$\frac{1}{8}$
$\frac{1}{8}$
$\frac{1}{8}$

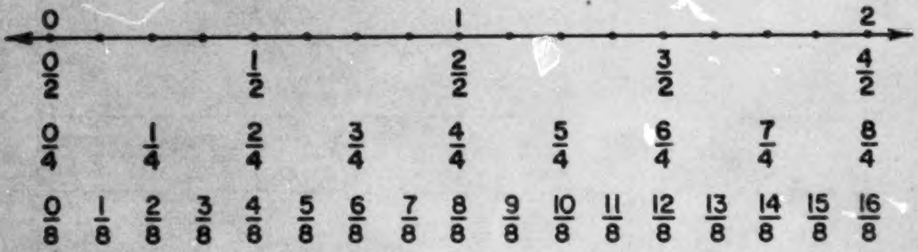

$\frac{1}{2}$
$\frac{1}{4}$
$\frac{1}{4}$


$\frac{1}{8}$
$\frac{1}{2}$
$\frac{1}{8}$
$\frac{1}{8}$
$\frac{1}{8}$


$\frac{1}{4}$
$\frac{1}{8}$
$\frac{1}{8}$
$\frac{1}{8}$
$\frac{1}{8}$
$\frac{1}{4}$

## ORDERING THE RATIONAL NUMBERS

### Exploration



Look at the number line.

Is $\frac{1}{2}$ to the right of $\frac{1}{4}$? Is $\frac{1}{2} > \frac{1}{4}$?

Is $\frac{3}{4}$ to the right of $\frac{3}{8}$? Is $\frac{3}{4} > \frac{3}{8}$?

Is $\frac{5}{4}$ to the right of $\frac{4}{2}$? Is $\frac{5}{4} > \frac{4}{2}$?

Is 0 to the left of $\frac{1}{4}$? Is $0 < \frac{1}{4}$?

Is $\frac{2}{4}$ to the left of $\frac{4}{2}$? Is $\frac{2}{4} < \frac{4}{2}$?

Is $\frac{5}{8}$ to the left of $\frac{6}{8}$? Is $\frac{5}{8} < \frac{6}{8}$?

It is easy to see that $\frac{1}{4}$, $\frac{3}{4}$, $\frac{6}{4}$, and $\frac{8}{4}$ are ordered from least to greatest.

Are $\frac{1}{2}$, $\frac{5}{8}$, $\frac{5}{4}$, and $\frac{4}{2}$ ordered from the least to greatest?

It would be easier to decide if we used other fractions for these numbers.

Using other names for these same numbers, we can write them as $\frac{4}{8}$, $\frac{5}{8}$, $\frac{10}{8}$, and $\frac{16}{8}$.

Now we see the numbers are named in order from least to greatest.

As you move to the right along a number line, the rational numbers become greater. As you move to the left, they become less.

## Exercise Set 6

1. Use this chart and the symbols > and < to complete the sentences below.

<table>
<tr><td colspan="8">$\frac{1}{2}$</td><td colspan="8">$\frac{1}{2}$</td></tr>
<tr><td colspan="4">$\frac{1}{4}$</td><td colspan="4">$\frac{1}{4}$</td><td colspan="4">$\frac{1}{4}$</td><td colspan="4">$\frac{1}{4}$</td></tr>
<tr><td colspan="2">$\frac{1}{8}$</td><td colspan="2">$\frac{1}{8}$</td><td colspan="2">$\frac{1}{8}$</td><td colspan="2">$\frac{1}{8}$</td><td colspan="2">$\frac{1}{8}$</td><td colspan="2">$\frac{1}{8}$</td><td colspan="2">$\frac{1}{8}$</td><td colspan="2">$\frac{1}{8}$</td></tr>
<tr><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td></tr>
</table>

a) $\frac{1}{2}$ ____ $\frac{1}{4}$

b) $\frac{1}{8}$ ____ $\frac{1}{4}$

c) $\frac{3}{8}$ ____ $\frac{1}{4}$

d) $\frac{1}{2}$ ____ $\frac{3}{8}$

e) $\frac{3}{4}$ ____ $\frac{5}{8}$

f) $\frac{3}{8}$ ____ $\frac{2}{4}$

2. Write the correct answer. The fraction chart above may be used, if needed.

a) Which number is less: $\frac{17}{8}$ or $\frac{16}{8}$ ?

Which is farther to the left on the number line?

b) Which number is less: $\frac{10}{8}$ or $\frac{12}{8}$ ?

Which is farther to the left on the number line?

c) Which number is less: $\frac{17}{8}$ or $\frac{15}{8}$ ?

Which is farther to the left on the number line?

d) Which number is less: $\frac{11}{4}$ or $\frac{4}{2}$ ?

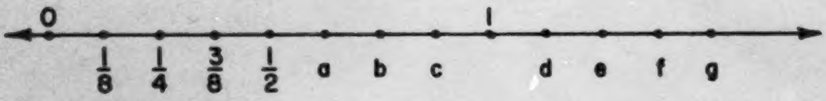
Which is farther to the left on the number line?

3. Arrange members of each set in order from least to greatest. Make diagrams if you need them.

$$A = \{ \frac{7}{2}, \frac{3}{2}, \frac{11}{2}, \frac{13}{2}, \frac{5}{2} \}$$

$$B = \{ \frac{7}{4}, \frac{3}{4}, 2, \frac{9}{4}, \frac{11}{4} \}$$

4. Associate a rational number with points a, b, c, d, e, f, and g in the diagram below.



a =

c =

e =

g =

b =

d =

f =

5. List in order the numbers used in counting by two-thirds from $\frac{2}{3}$ to 4.

6. List in order the numbers used in counting by three-halves from $\frac{3}{2}$ to 9.

7. Write two other names for each of the following numbers.

a) $\frac{12}{8}$

c) $\frac{10}{4}$

b) $\frac{5}{2}$

d) 3

8. Copy and complete by writing the symbol > or < in each box.

a) $\frac{1}{4}$ ____ $\frac{1}{2}$

b) $\frac{1}{2}$ ____ $\frac{1}{8}$

c) $\frac{1}{10}$ ____ 1

d) 1 ____ $\frac{1}{2}$

e) $\frac{1}{4}$ ____ $\frac{1}{8}$

f) $\frac{1}{6}$ ____ $\frac{1}{3}$

9. Rearrange these numbers in order from least to greatest.

a) $\frac{5}{8}$, $\frac{3}{8}$, $\frac{3}{4}$, $\frac{1}{8}$, $\frac{1}{4}$

b) $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{2}$, $\frac{1}{6}$, $\frac{1}{4}$

c) $\frac{2}{3}$, $\frac{5}{6}$, $\frac{1}{6}$, $\frac{1}{3}$

d) $\frac{3}{8}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{5}$

10. Arrange in order the numbers in each set below. Begin with the greatest.

$A = \left\{\frac{2}{4}, \frac{3}{4}, \frac{1}{4}\right\}$

$B = \left\{\frac{1}{4}, \frac{1}{2}, \frac{1}{8}\right\}$

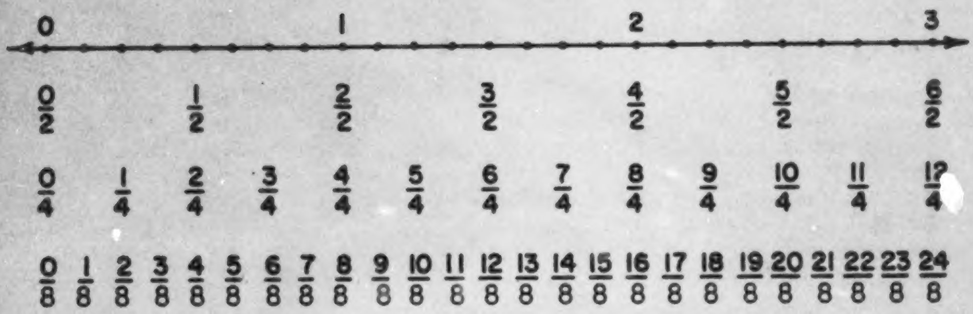
$C = \left\{\frac{1}{2}, \frac{3}{8}, \frac{3}{4}\right\}$

11. Arrange these numbers from least to greatest.

$\frac{1}{2}$, $\frac{1}{10}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{3}$, $\frac{1}{6}$.

## Exercise Set 7

### Supplementary Exercises


0 1 2 3
0/2 1/2 2/2 3/2 4/2 5/2 6/2
0/4 1/4 2/4 3/4 4/4 5/4 6/4 7/4 8/4 9/4 10/4 11/4 12/4
0/8 1/8 2/8 3/8 4/8 5/8 6/8 7/8 8/8 9/8 10/8 11/8 12/8 13/8 14/8 15/8 16/8 17/8 18/8 19/8 20/8 21/8 22/8 23/8 24/8

1. Copy and write $>$, $<$, or $=$ in each blank to make a true sentence. The number line above will help you.

a) $\frac{5}{4}$ ____ $\frac{1}{2}$

b) $\frac{8}{4}$ ____ $2$

c) $3$ ____ $\frac{6}{2}$

d) $\frac{3}{2}$ ____ $\frac{3}{4}$

e) $\frac{2}{2}$ ____ $1$

f) $\frac{3}{4}$ ____ $\frac{5}{8}$

g) $\frac{7}{4}$ ____ $\frac{11}{8}$

h) $\frac{18}{8}$ ____ $\frac{8}{4}$

2. Which fraction of each pair below will be farther to the right on the number line?

a) $\frac{19}{8}$ or $\frac{17}{8}$

b) $\frac{11}{5}$ or $\frac{5}{4}$

c) $\frac{5}{2}$ or $\frac{18}{8}$

d) $\frac{10}{2}$ or $\frac{5}{2}$

e) $\frac{14}{8}$ or $\frac{6}{4}$

f) $\frac{11}{4}$ or $\frac{4}{2}$

g) $\frac{5}{4}$ or $\frac{3}{2}$

h) $\frac{1}{2}$ or $\frac{1}{4}$

3. Rearrange each set. Put members in order from least to greatest.

$A = \left\{\frac{7}{2}, \frac{3}{2}, \frac{11}{2}, \frac{5}{2}\right\}$

$B = \left\{\frac{1}{4}, \frac{1}{2}, \frac{1}{3}, \frac{1}{8}\right\}$

4. Copy and fill in each blank with the symbol >, <, or = .

a) $\frac{3}{3}$ ____ $\frac{1}{3}$

b) $2$ ____ $\frac{4}{3}$

c) $\frac{4}{4}$ ____ $1$

d) $\frac{4}{5}$ ____ $\frac{2}{5}$

e) $1$ ____ $2$

f) $\frac{3}{3}$ ____ $\frac{2}{2}$

g) $2$ ____ $\frac{7}{4}$

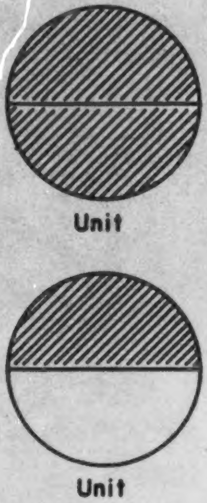
h) $\frac{1}{2}$ ____ $1$

5. Look at exercise 4. Which fraction in each pair labels a point farther to the right on the number line?

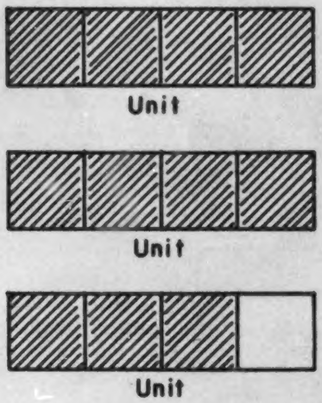
# A NEW KIND OF NAME

These pictures help us think about the numbers, $\frac{3}{2}$ and $\frac{11}{4}$.

A.



B.



$\frac{3}{2} = \frac{2}{2}$ and $\frac{1}{2}$

$\frac{3}{2} = 1$ one and 1 half

or,

$\frac{3}{2} = 1\frac{1}{2}$

$\frac{11}{4} = \frac{4}{4}$ and $\frac{4}{4}$ and $\frac{3}{4}$

$\frac{11}{4} = 1$ one and 1 one
and 3 fourths

$\frac{11}{4} =$ 2 ones and
3 fourths

or,

$\frac{11}{4} = 2\frac{3}{4}$

Another way of naming $\frac{3}{2}$ is $1\frac{1}{2}$

Another way of naming $\frac{11}{4}$ is $2\frac{3}{4}$

We call $1\frac{1}{2}$ and $2\frac{3}{4}$ mixed forms.

Rational numbers named by fractions like $\frac{1}{2}$, $\frac{3}{4}$, $\frac{2}{3}$, and $\frac{7}{8}$ tell us that the measure of a region, segment, or set is less than 1.

Rational numbers named by fractions like $\frac{2}{2}$, $\frac{8}{8}$, $\frac{4}{4}$, and $\frac{3}{3}$ tell us that the measure of a region, segment, or set is equal to 1.

Rational numbers named by fractions like $\frac{7}{4}$, $\frac{3}{2}$, and $\frac{5}{3}$ tell us that the measure of a region, segment, or set is greater than 1.

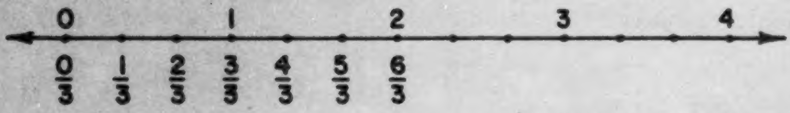
Other names for 1 are $\frac{4}{4}$, $\frac{2}{2}$, and $\frac{3}{3}$.

Since this is true, $\frac{7}{4}$, $\frac{3}{2}$, and $\frac{5}{3}$ may be renamed $1\frac{3}{4}$ $1\frac{1}{2}$, and $1\frac{2}{3}$.

$1\frac{3}{4}$, $1\frac{1}{2}$, and $1\frac{2}{3}$ are read, "one and three-fourths," "one and one-half," and "one and two-thirds." Fractions written in this way are said to be in mixed form.

## Exercise Set 8

1. Copy and finish the number line below. Then use it to complete the mathematical sentences so that each will be a true sentence.



a) $\frac{4}{3} = \frac{\_\_}{3}$ and $\frac{1}{3}$

$\frac{4}{3} = \_\_\_\_\_\_$

b) $\frac{6}{3}$ is $\frac{\_\_}{3}$ and $\frac{3}{3}$

$\frac{6}{3} = \_\_$

c) $\frac{11}{3} = 3$ ones and $\frac{\_\_}{3}$

$\frac{11}{3} = 3\frac{\_\_}{3}$

d) $2\frac{2}{3} = 2$ and $\frac{2}{3}$

$2\frac{2}{3} = \frac{6}{3}$ and $\frac{2}{3}$

$2\frac{2}{3} = \frac{\_\_}{3}$

e) $4 = \frac{\_\_}{3}$

f) $3\frac{1}{3} = \frac{9}{3}$ and $\frac{\_\_}{3}$

$3\frac{1}{3} = \_\_\_\_\_\_$

2. Arrange the numbers in each of the following sets in order from least to greatest. Use diagrams if you need them.

$A = \left\{\frac{3}{4},\ 0,\ \frac{7}{4},\ 2,\ \frac{9}{4},\ 1\right\}$

$B = \left\{\frac{5}{3},\ 1,\ \frac{10}{3},\ 4,\ \frac{7}{3},\ 2,\ \frac{2}{3},\ 3\right\}$

3. Peter has 13 blocks to walk to school. Each block is $\frac{1}{10}$ mile long. How many miles does he have to walk to school?

4. A pound of butter is usually divided into four bars of the same size. Vicky found 7 bars of butter in her refrigerator. How many pounds of butter were in the refrigerator?

Can you do these without any help? Try some of them.

5. Write the mixed form for each of these numbers.

a) $\frac{5}{4} =$ d) $\frac{9}{2} =$

b) $\frac{6}{2} =$ e) $\frac{12}{5} =$

c) $\frac{8}{5} =$ f) $\frac{7}{3} =$

6. Which is greater? Write the name of the greater number in each pair. You may use a number line to help you decide.

a) $\frac{5}{3}$ or $1\frac{1}{2}$ e) $\frac{8}{7}$ or $1\frac{1}{8}$

b) $2\frac{3}{4}$ or $\frac{10}{4}$ f) $3\frac{1}{6}$ or $\frac{5}{2}$

c) $\frac{12}{3}$ or $3\frac{7}{8}$ g) $4\frac{2}{3}$ or $\frac{31}{10}$

d) 6 or $\frac{21}{3}$ h) $1\frac{3}{4}$ or $\frac{8}{4}$

7. Copy and complete. Use diagrams if you need them.

a) $1\frac{2}{5} = \frac{\phantom{0}}{5}$ d) $3\frac{1}{2} = \frac{\phantom{0}}{2}$

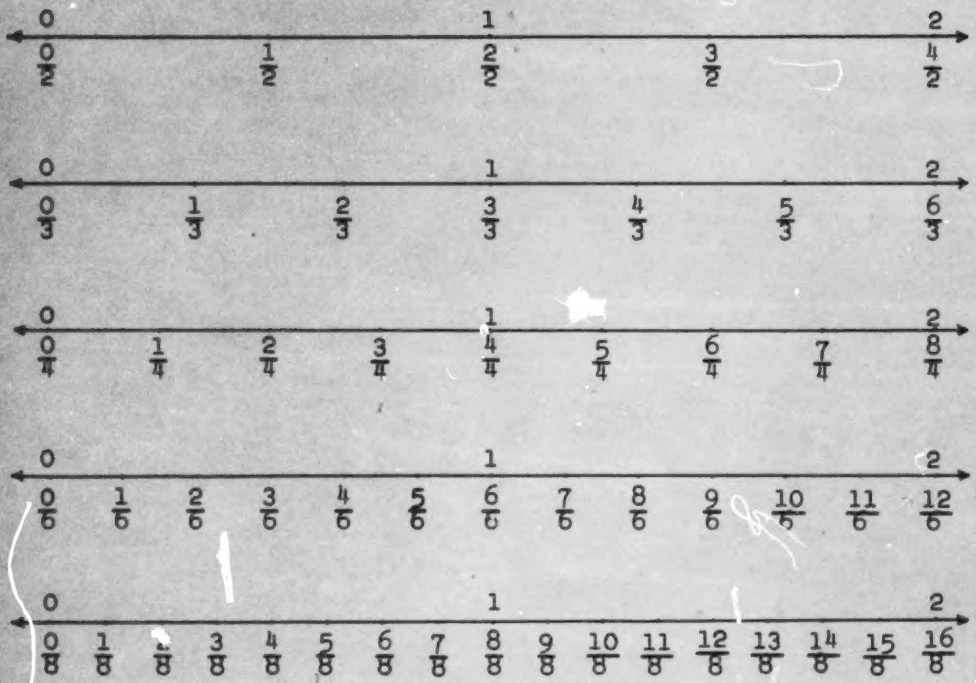
b) $2\frac{1}{3} = \frac{\phantom{0}}{3}$ e) $1\frac{5}{6} = \frac{11}{\phantom{0}}$

c) $2\frac{3}{4} = \frac{\phantom{0}}{4}$ f) $2\frac{3}{8} = \frac{19}{\phantom{0}}$

8. Between what two whole numbers on the number line would the following fractions be?

a) $5\frac{1}{3}$ b) $2\frac{3}{4}$ c) $7\frac{1}{2}$ d) $6\frac{9}{10}$

## Exercise Set 9


0 1 2
0/2 1/2 2/2 3/2 4/2
0 1 2
0/3 1/3 2/3 3/3 4/3 5/3 6/3
0 1 2
0/4 1/4 2/4 3/4 4/4 5/4 6/4 7/4 8/4
0 1 2
0/6 1/6 2/6 3/6 4/6 5/6 6/6 7/6 8/6 9/6 10/6 11/6 12/6
0 1 2
0/8 1/8 2/8 3/8 4/8 5/8 6/8 7/8 8/8 9/8 10/8 11/8 12/8 13/8 14/8 15/8 16/8

1. Use the number lines above. Copy the following mathematical sentences. Write the symbol > or < in each blank to make the sentence true.

a) $\frac{10}{8}$ ____ $\frac{12}{8}$

b) $2$ ____ $\frac{3}{4}$

c) $\frac{5}{6}$ ____ $1$

d) $\frac{3}{2}$ ____ $\frac{5}{4}$

e) $\frac{3}{6}$ ____ $\frac{9}{8}$

f) $\frac{8}{8}$ ____ $\frac{4}{3}$

g) $\frac{17}{4}$ ____ $\frac{11}{8}$

h) $\frac{4}{3}$ ____ $\frac{7}{6}$

2. Starting at zero, list in order the numbers used in

a) counting by one-half to $\frac{4}{2}$

b) counting by two-thirds to $\frac{6}{3}$

c) counting by three eighths to $\frac{15}{8}$

3. Write 2 other names for each of the following.

a) $\frac{1}{2}$ = ____, ____

b) 1 = ____, ____

c) $1\frac{1}{4}$ = ____, ____

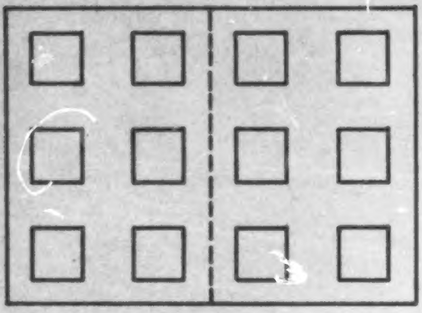
d) $2\frac{1}{2}$ = ____, ____

4. Match each rational number in Column 1 with a fraction that names the same number from Column 2.

| | Column 1 | | Column 2 |
|---|---|---|---|
| a) | $\frac{1}{2}$ | f) | $\frac{8}{4}$ |
| b) | $\frac{1}{4}$ | g) | $\frac{6}{4}$ |
| c) | 2 | h) | $\frac{2}{4}$ |
| d) | $1\frac{1}{2}$ | i) | $\frac{2}{2}$ |
| e) | 1 | j) | $\frac{2}{8}$ |

## Exploration

Below are pictures of sets of 12 objects.



Set A

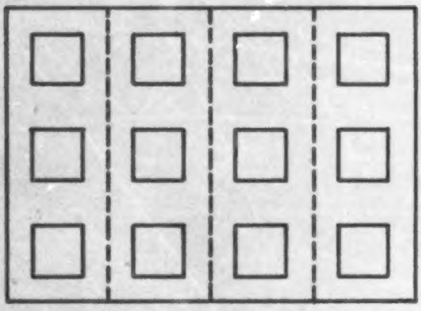
Dotted lines separate the picture of Set A into 2 subsets.

How many objects are there in 1 subset?

How many objects are there in 2 subsets?

Is $\frac{1}{2}$ of 12 objects equal to 6 objects?

Is $\frac{2}{2}$ of 12 objects equal to 12 objects?



Set B

Set B has been separated into 4 subsets.

How many objects are in each subset?

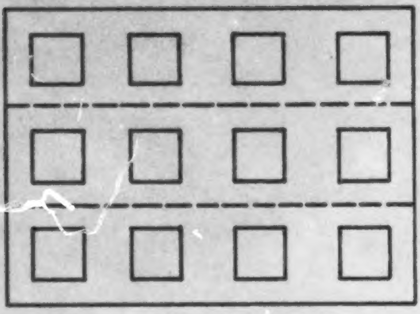
$\frac{1}{4}$ of 12 = ______

$\frac{2}{4}$ of 12 = ______

$\frac{3}{4}$ of 12 = ______

$\frac{4}{4}$ of 12 = ______

Is $\frac{1}{2}$ of 12 = $\frac{2}{4}$ of 12?



Set C

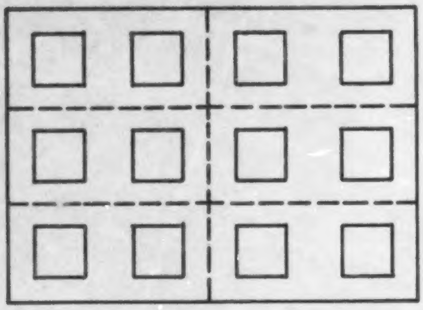
Dotted lines separate Set C into

______ subsets.

What is $\frac{1}{3}$ of 12?

What is $\frac{2}{3}$ of 12?

What is $\frac{3}{3}$ of 12?



Set D

Set D has been separated into

______ subsets.

2 = $\frac{\quad}{6}$ of 12.

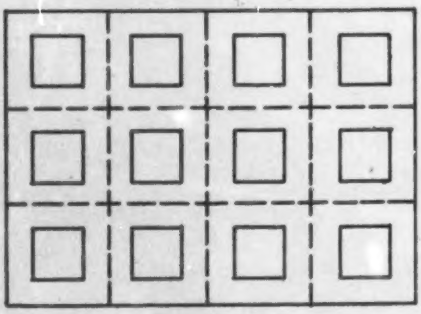
4 = $\frac{\quad}{6}$ of 12.

6 = $\frac{\quad}{6}$ of 12.

8 = $\frac{\quad}{6}$ of 12.

10 = $\frac{\quad}{6}$ of 12.

12 = $\frac{\quad}{6}$ of 12.



Set E

Each subset in E shows

______ of 12.

$\frac{3}{12}$ of 12 = ____

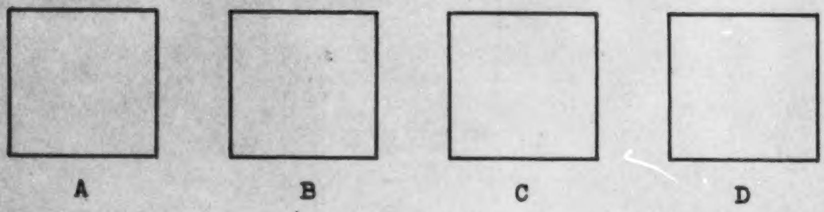
$\frac{4}{12}$ of 12 = ____

$\frac{6}{12}$ of 12 = ____

$\frac{8}{12}$ of 12 = ____

$\frac{9}{12}$ of 12 = ____

## Exercise Set 10



1. A, B, C, and D are unit square regions. Copy them on your paper. Separate each one into four equal regions.

a) Color $\frac{1}{4}$ of A red.

b) Color $\frac{2}{4}$ of B blue.

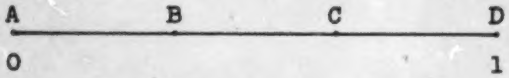
c) Color $\frac{3}{4}$ of C green.

d) Color $\frac{4}{4}$ of D green.

e) $\frac{4}{4}$ is another name for ______.

f) Write the fraction that best describes the uncolored regions of each unit square region above.

2.



Points B and C separate the unit line segment AD into 3 congruent segments.

a) $m\,\overline{AB} =$

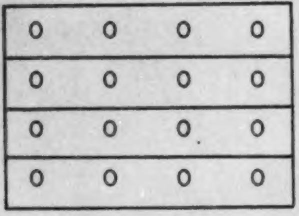
c) $m\,\overline{AD} =$

b) $m\,\overline{AC} =$

d) $\frac{3}{3} =$ ______

## Exercise Set 11

1. Look at the picture of a set of objects below.



It has been partitioned into 4 subsets.

The same number of objects is in each subset.

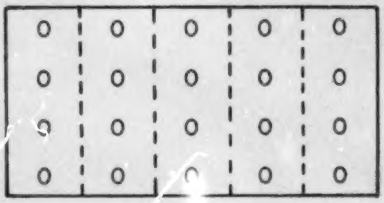
What is $\frac{1}{4}$ of 16?

What is $\frac{2}{4}$ of 16?

What is $\frac{3}{4}$ of 16?

What is $\frac{4}{4}$ of 16?

2. Here is another picture of a set of objects. It has been partitioned into five subsets. The same number of objects is in each subset.



What is $\frac{1}{5}$ of 20?

What is $\frac{2}{5}$ of 20?

What is $\frac{3}{5}$ of 20?

What is $\frac{4}{5}$ of 20?

What is $\frac{5}{5}$ of 20?

3. Complete the following. Use sets of objects if you need them.

a) $\frac{1}{3}$ of 6 is ______

b) $\frac{1}{2}$ of 4 is ______

c) $\frac{1}{4}$ of 8 is ______

d) $\frac{1}{2}$ of 10 is ______

e) $\frac{2}{3}$ of 6 is ______

f) $\frac{3}{4}$ of 8 is ______

g) $\frac{2}{3}$ of 9 is ______

h) $\frac{2}{5}$ of 10 is ______

4. Jane bought six doughnuts. She ate $\frac{1}{3}$ of them. How many doughnuts did Jane eat? How many doughnuts did Jane have left?

5. Bill had twenty marbles. He lost $\frac{1}{4}$ of them. How many marbles did Bill lose? How many did he have left?

6. Alice had 36 jacks. She traded $\frac{1}{4}$ of them to Mary. How many jacks did Alice trade? How many jacks did Alice have left?

7. On the way from the store, Bob dropped a dozen eggs. He looked inside the carton. He found $\frac{3}{4}$ of the eggs broken. How many eggs are there in a dozen? How many eggs were broken? How many eggs were not broken?

BRAINTWISTER

John gave Bill sixteen jelly beans. This was $\frac{1}{2}$ of the number John had. How many did John have at the beginning?

## Exercise Set 12

1. There were 20 problems on an arithmetic test. John worked all but $\frac{1}{4}$ of them. How many problems did John finish?

2. $\frac{1}{3}$ of a string of 12 Christmas tree lights had burned out. How many lights had to be replaced?

3. At a sale, books that had been 50¢ were selling for $\frac{1}{2}$ of the regular price. What was the sale price?

4. A box which had contained 24 candy bars was two-thirds full. How many candy bars were in the box?

5. A football game is played in 4 quarters. It takes 1 hour of actual playing time to play a game. How many minutes of actual playing time are gone at the end of the third quarter?

6. There were 6 boys and 3 girls on a softball team. What part of the team was boys?

7. The year is separated into four seasons of equal length. What part of the year is each season?

8. Mary has a collection of 15 dolls. $\frac{2}{3}$ of them represent children from other countries. How many of the dolls represent children from other countries?

9. Jim was making a model of a plane. He needed a single piece of wood $\frac{3}{4}$ of a foot long. He had a piece of wood 8 inches long. Could he use this piece? Why?

## Practice Exercises

1. Place parentheses correctly to make each of the following a true statement. Example a is shown.

a) $(6 + 4) \times 3 = 30$

b) $8 \times 3 + 5 < 64$

c) $6 + 3 \times 6 > 24$

d) $2 \times 5 + 4 = 18$

e) $4 \times 16 + 4 > 68$

f) $9 + 6 \div 3 = 11$

g) $8 \times 5 + 3 \neq 43$

h) $6 \div 3 \times 4 = 8$

i) $18 \div 6 + 3 \neq 2$

j) $14 \div 7 + 7 > 1$

k) $5 \times 8 - 2 > 30$

l) $25 \div 5 + 8 = 13$

m) $19 + 8 \div 2 > 14$

n) $45 \div 5 + 4 < 13$

o) $27 + 3 \div 6 = 5$

p) $28 - 7 \times 3 < 63$

q) $46 + 8 \div 9 = 6$

r) $28 + 21 \div 7 \neq 7$

s) $17 - 4 \times 3 < 39$

t) $49 \div 7 + 6 = 13$

2. Mixed Addition and Subtraction

a) 327 + 54

b) 457 + 218

c) 384 + 291

d) 384 - 156

e) 995 - 768

f) 870 - 418

g) 2384 - 1963

h) 1066 - 883

i) 984 + 168

j) 700 - 362

k) 1478 + 2388

l) 400 + 583 + 324

m) 1637 - 537

n) 709 - 368

o) 37 + 31 + 36

p) 801 - 513

q) 745 - 508

r) 678 + 254

s) 2900 - 1256

t) 598 + 303 + 81

3. Write the number n represents.

a) 29 + 56 + 37 = n
b) 700 - 347 = n
c) 43 x 6 = n
d) 587 - n = 369
e) 77 + 94 + n = 237
f) n x 6 = 3708
g) 48 + n + 79 = 234
h) n = 127 x 5
i) 746 - n = 413
j) 624 + n = 1141
k) n = 737 x 8
l) n + 304 + 488 = 1640
m) 4767 = n x 7
n) 719 - n = 285
o) 8789 + n = 12497
p) 707 x 6 = n
q) 8789 - n = 5081
r) 489 + 403 + 950 = n
s) n x 9 = 7857
t) n - 658 = 758

4. Addition, Subtraction, Multiplication, and Division.

a) 1414 - 671
b) 2157 + 879
c) 148 ÷ 4
d) 367 x 6
e) 459 ÷ 9
f) 309 + 487 + 648
g) 475 - 367
h) 280 ÷ 7
i) 396 x 7
j) 1209 - 688
k) 278 + 32 + 49
l) 378 ÷ 6
m) 439 x 5
n) 679 - 327
o) 136 ÷ 4
p) 810 + 652 + 934
q) 333 x 7
r) 652 - 584
s) 444 ÷ 6
t) 876 x 4

5. Write the number n represents

a) $\frac{1}{2}$ of 12 = n
b) $\frac{1}{3}$ of 15 = n
c) $\frac{1}{4}$ of 8 = n
d) $\frac{1}{9}$ of 9 = n
e) $\frac{1}{5}$ of 20 = n
f) $\frac{1}{6}$ of 18 = n
g) $\frac{2}{3}$ of 9 = n
h) $\frac{3}{4}$ of 8 = n
i) $\frac{2}{5}$ of 10 = n
j) $\frac{1}{8}$ of 16 = n
k) $\frac{4}{5}$ of 15 = n
l) $\frac{3}{8}$ of 16 = n

6. Find the unknown addend by regrouping.

Example:

$$\begin{array}{rcrcl} 462 & = & 400 + 60 + 2 & = & 400 + 50 + 12 \\ \underline{-157} & = & 100 + 50 + 7 & = & \underline{100 + 50 + \ \ 7} \\ & & & & 300 + 00 + \ \ 5 \ = \ 305 \end{array}$$

a) 609
$\underline{-362}$

b) 633
$\underline{-563}$

c) 386
$\underline{-219}$

d) 890
$\underline{-437}$

e) 638
$\underline{-394}$

f) 853
$\underline{-628}$

g) 493
$\underline{-316}$

h) 761
$\underline{-257}$

i) 780
$\underline{-333}$

j) 956
$\underline{-247}$

7. **Multiplication and Division**

a) 72 ÷ 8
b) 789 × 5
c) 725 ÷ 5
d) 864 × 6
e) 408 ÷ 4
f) 904 × 7
g) 824 ÷ 4
h) 496 × 6
i) 654 ÷ 3
j) 730 × 9
k) 378 ÷ 3
l) 257 × 4
m) 2488 ÷ 8
n) 319 × 8
o) 580 ÷ 5
p) 509 × 7
q) 789 ÷ 3
r) 156 × 9
s) 217 ÷ 7
t) 697 × 3

8. Write the numeral for each blank that makes a true sentence. Example a is shown.

a) ( 4 ÷ 2) × 9 = 18
b) (7 × ___) - 29 = 34
c) (8 × 8) - ___ = 50
d) (12 × 6) - 65 = ___
e) (___ × 8) - 39 = 9
f) (4 × ___) + 6 = 30
g) (6 × 6) - ___ = 14
h) (54 - 47) × ___ = 49
i) (11 - 5) × 9 = ___
j) (8 + 20) - ___ = 14
k) (___ × 4) + 8 = 44
l) (6 × ___) - 15 = 21
m) (7 × 8) + 16 = ___
n) (8 × ___) - 12 = 28
o) (7 × 9) + 12 = ___
p) (5 × ___) - 14 = 21
q) (___ × 9) - 6 = 48
r) (7 × 6) + ___ = 55
s) (___ × 6) - 7 = 29
t) (7 × 9) + ___ = 72

## Review

## SET I

### Part A

1. Using the symbol $>$ , $=$ , or $<$ make each of the following a true sentence.

a) $\frac{2}{3}$ foot ____ 12 inches

b) 24 inches ____ $\frac{1}{2}$ yard

c) 1 pint ____ $\frac{1}{2}$ quart

d) $\frac{1}{2}$ hour ____ 30 minutes

e) $\frac{2}{3}$ yard ____ 2 feet

f) 12 inches ____ $\frac{1}{4}$ yard

g) $\frac{1}{2}$ quart ____ 2 pints

h) 15 minutes ____ $\frac{1}{4}$ hour

i) 4 feet ____ $1\frac{1}{2}$ yards

j) 9 inches ____ $\frac{1}{3}$ yard

2. Arrange in order of size from smallest to largest.

a) $\frac{2}{3}$ , $\frac{1}{3}$ , $\frac{6}{3}$ , $\frac{4}{3}$ , $\frac{8}{3}$

b) $\frac{1}{2}$ , $\frac{1}{3}$ , $\frac{1}{7}$ , $\frac{1}{5}$ , $\frac{1}{9}$

c) $\frac{3}{6}$ , $\frac{3}{3}$ , $\frac{3}{4}$ , $\frac{3}{8}$ , $\frac{3}{12}$

d) $\frac{1}{2}$ , $\frac{3}{4}$ , $\frac{2}{8}$ , $\frac{4}{4}$ , $\frac{1}{8}$

e) $\frac{4}{5}$ , $\frac{3}{5}$ , $\frac{1}{5}$ , $\frac{2}{5}$ , $\frac{7}{5}$

f) 1 ft., 6 in., $\frac{3}{4}$ ft., $\frac{1}{4}$ ft.

g) 45 min., $\frac{1}{2}$ hr., 60 min., $\frac{1}{4}$ hr.

h) 1 ton, 1 lb., $\frac{1}{2}$ lb., 1 oz..

i) 1 pt., 1 qt., 1 cup., 1 gal..

j) $\frac{1}{3}$ in., $\frac{2}{3}$ in., $\frac{5}{6}$ in., $\frac{1}{6}$ in..

3. N = {The named points A, B}

   What is the greatest number of line segments that can have endpoints in Set N ?

   Name the segment(s).

4. M = {The named points C, D, E}

   What is the greatest number of line segments that can have endpoints in Set M ?

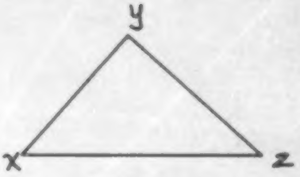
   Name the segment(s).

5. R = {The named points G, F, H, I}

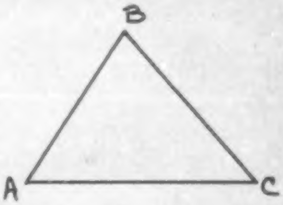
   What is the greatest number of line segments that can have endpoints in Set R ?

   Name the segment(s).

6. Draw two line segments to make three quadrilaterals and five new triangles out of △ XYZ.



7. Draw two line segments to make two new triangles and three quadrilaterals out of △ ABC.



8. $4 < 5 < 8$ means 4 is less than 5 and 5 is less than 8.

   Write these sentences the shorter way.

   a) $32 < 34$ and $34 < 40$
   b) $\frac{1}{4} < \frac{2}{4}$ and $\frac{2}{4} < \frac{3}{4}$
   c) $112 < 115$ and $115 < 117$
   d) $\frac{1}{2} < \frac{3}{4}$ and $\frac{3}{4} < \frac{7}{8}$

9. Find the number that n represents in the following:

a) $14 < n < 16$

b) $\frac{1}{3} < n < \frac{3}{3}$

c) $786 < n < 788$

d) $\frac{3}{8} < n < \frac{5}{8}$

e) $9 < n < \frac{20}{2}$

f) $\frac{1}{2} < n < 1\frac{1}{2}$

10. What is it?

a) A model that has 3 rectangular regions and 2 triangular regions for faces.

b) A model that has four triangular regions for faces.

c) A model that has six rectangular regions for faces.

d) A model that has one rectangular region and two circular regions for faces.

e) A model whose edges form right angles only.

f) A model that has a circular region and a half circular region for faces.

11. These statements are comparing the length of line segments. Complete these to make them true statements. Examples a and b are done for you.

a) 4 ft. is $\underline{\ 2\ }$ times as long as 2 ft..

b) 2 ft. is $\underline{\ \frac{1}{2}\ }$ as long as 4 ft.

c) 9 in. is ___ times as long as 3 in..

d) 3 in. is ___ as long as 9 in..

e) ___yd. is 2 times as long as 6 yd..

f) 6 yd. is ___ as long as 12 yd..

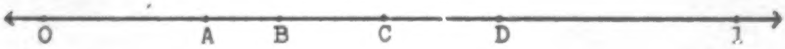
g) 15 min. is ___ as long as 45 min.

h) 45 min. is ___ times as long as 15 min..

12. Match the Standard Unit of Measure from Column I with the Item you would use it to measure from Column II

| Column I | | Column II |
|---|---|---|
| a) cup | _____ | Length of your pencil |
| b) feet | _____ | Bottle of milk |
| c) seconds | _____ | Distance between cities |
| d) hours | _____ | Length of a football field |
| e) days | _____ | Timing a running race |
| f) quart | _____ | Time at recess |
| g) yards | _____ | Time until a birthday |
| h) minutes | _____ | Time for sleeping |
| i) miles | _____ | Height of a tree |
| j) inches | _____ | Sugar for a recipe |

13. Write 4 different fraction names for each of the points labeled on this number line.



14. Find the perimeter of the following:

a) A polygon with sides whose measures are 16, 28, and 32 in inches.

b) An equilateral triangle with the measure of one of its sides 14 in feet.

c) A polygon with 6 congruent sides, the measure of one side is 35 in centimeters.

d) A square, one side of which has the measure of 7 in meters

e) A polygon with 2 sides whose measures have the sum of 8 yards and 3 sides whose measures have the sum of 15 in yards.

Part B

Write a mathematical sentence (or two sentences if necessary) for each problem and solve. Write an answer sentence.

1. The Davis family uses 8 eggs for breakfast. What part of a dozen eggs is left?

2. Eddie earns 75 cents on Monday, 50 cents on Wednesday, and 75 cents on Friday mowing lawns. How much will he earn in six weeks?

3. The school bus runs 7 miles on a gallon of gasoline. Each week the bus goes an average of 882 miles. How much gasoline will the bus use in four weeks?

4. Wendy called the feed store to order feed for a month for her horse. She bought 5 bales of hay at $1.75 a bale and 100 lb. of oats at $5.30. How much will this month's feed bill be?

5. Tom hit a softball 135 ft. Randy hit the ball 25 ft. farther than Tom. How far did Randy hit the ball?

6. It is 347 air miles from San Francisco to Los Angeles, 1240 air miles on to Dallas, 443 air miles from Dallas to New Orleans, then 669 air miles on to Miami. How many air miles is it by this route, from San Francisco to Miami?

7. The nurse found Janice to be 4 ft. 4 in. tall, Linda 4 ft. 11 in. tall, and Maria 4 ft. 9 in. tall. How much taller than Janice is Linda?

8. In the standing broad-jump Pat's best jump was 5 ft. 3in. while Roy's best jump was 6 ft. 2 in. Roy's jump was how much better than Pat's?

9. For his birthday Tom received a new baseball bat that is 24 inches long. The bat's length is what part of a yard?

10. Joe delivers 56 papers each day. How many papers does he deliver in 28 days?

11. Susan buys 2 dozen cookies for 30 cents a dozen and a cake for 80 cents. How much does she pay the clerk?

## Braintwisters

1. You have a 30 inch board that you have to cut in 5 pieces, each 6 inches long. It takes five minutes to make each cut. How many minutes will it take you to cut the 5 pieces?

2. An inchworm was climbing a tree 5 feet high. He climbed three inches every day and slipped back two inches every night. How many days will it take him to reach the top?

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The following is a list of all those who have participated in the preparation of this volume.

Truman A. Botts, University of Virginia
James A. Cooley, University of Tennessee
Helen L. Curran, Glenview School, Oakland, California
Helen L. Garstens, University of Maryland
E. Glenadine Gibb, State College of Iowa
Geraldine Green, Vetal School, Detroit, Michigan
William T. Guy, Jr., University of Texas
Leon Haaland, Kenwood School, Minneapolis, Minnesota
Clarence Ethel Hardgrove, Northern Illinois University
Royce S. Hargrove, Corpus Christi Public Schools, Texas
Max Hosier, State College of Iowa
Henry G. Jacob, University of Massachusetts
Lenore John, University High School, University of Chicago
George E. Knoblock, El Carmelo School, Palo Alto, California
William G. Lister, State University of New York
Lelia M. Maneely, Springer School, Los Altos, California
John L. Marks, San Jose State College
Martha Meek, Glorietta Elementary School, Orinda, California
Mary McDermott, Mt. Diablo Unified School District, California
William K. McNabb, St. Mark's School, Dallas, Texas
Frances J. Mettler, Walter Hays Elementary School, Palo Alto, California
Leon Rutland, University of Colorado
Irene Sauble, Detroit Public Schools
Helen Schneider, LaGrange Public Schools, Illinois
Willa J. Sessions, Hillsborough County Public Schools, Florida
Rose Mary Shea, Edith C. Baker School, Brookline, Massachusetts
Wesley Thompson, Detroit Public Schools
Morgan Ward, California Institute of Technology
Ted Wassam, Ventura School, Palo Alto, California
J. Fred Weaver, Boston University